

Message Passing for Analysis and Resilient Design of Self-Healing Interdependent Cyber-Physical Networks

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Abstract—Coupling cyber and physical systems gives rise to numerous engineering challenges and opportunities. An important challenge is the contagion of failure from one system to another, that can lead to large scale cascading failures. On the other hand, *self-healing* ability emerges as a valuable opportunity where the overlay cyber network can cure failures in the underlying physical network. To capture both self-healing and contagion, we introduce a factor graph representation of inter-dependent cyber-physical systems in which factor nodes represent various node functionalities and the edges capture the interactions between the nodes. We develop a message passing algorithm to study the dynamics of failure propagation and healing in this representation. Through applying a fixed-point analysis to this algorithm, we investigate the network reaction to initial disruptions. Our analysis provides simple yet critical guidelines for choosing network parameters to achieve resiliency against cascading failures.

Index Terms—Cyber-Physical Systems, Factor Graph, Message Passing, Cascading Failure.

I. INTRODUCTION

A cyber-physical system can be define as a system of collaborating computational elements controlling physical entities. The future smart grid, intelligent transportation systems, distributed robotics, and medical monitoring systems are all examples of cyber-physical systems (CPS). The interconnected nature of such systems gives rise to numerous engineering challenges and opportunities. An important challenge is the contagion of failures from one system to another in a coupled system. Such contagions may lead to large scale catastrophic failures triggered by a small failure, such as the 2003 blackout in the Northeastern United States [1]. On the other hand, *self-healing* ability emerges as a valuable opportunity where the overlay cyber network can cure failures in the underlying physical network. For example, the traffic control network which monitors the taxi transportations could avoid congestions by calculating fastest routes in a given time of the day [2].

The study of interdependent networks was sparked by the seminal work of Buldyrev et al. [3], where a simple “one-to-one” interdependence model was considered. This work was continued in [4] with studying the percolation of failures after an attack, where the first and the second order transitions were demonstrated for randomly-coupled and scale-free networks. Parshani et al. in [5] proved that reduction of the coupling between the networks changes the percolation phase from first-order to second-order. A systematic strategy based on betweenness was introduced in [6] to select the minimum number of

autonomous nodes that guarantees a smooth transition, and was shown to reduce the fragility of network without losing functionality. Authors in [7] studied the impact of correlated inter-layer coupling on some models of real-world networks. Huang et al. in [8] developed a theoretical framework to study targeted attacks on nodes with a specific degree. It was shown that, unlike single-layer networks, protecting higher degree nodes in interdependent networks is not necessarily the best way to defend the network. A “regular allocation” algorithm was proposed in [9] to allocate the same number of inter-links to each node. It was proved that such allocation is optimal if the network topology is unknown. Authors in [10], [11] studied the influence of active small clusters appearing after an attack on the overall network performance. They particularly obtained an upper bound on the fraction of such clusters after a cascading failure.

The majority of the existing literature including the above applies percolation theory while focusing on the size of the remaining giant component after a cascading failure. On the other hand, self-healing, and its modeling and design advantages in cyber-physical systems have been mostly overlooked in the literature. Among the most important issues on the way to design the future cyber-physical systems such as the smart grid are to

- derive an analytically-tractable model that captures the key features of real-life systems such as self-healing and contagion,
- develop a framework that enables studying multiple layers of interconnected cyber and physical systems.

In this paper, we take a novel approach to address these issues by applying ideas inspired by error correction coding to model, analyze, and design cyber-physical systems. Our main contributions can be summarized as follows:

- 1) We propose a *factor graph* representation for cyber-physical systems, where factor nodes represent network functionalities of the cyber and physical nodes, and the edges capture the interactions between them. Both healing and contagion will be captured in this representation.
- 2) We extend the message passing algorithm used in low-density parity-check (LDPC) codes to the proposed factor graph representation. Utilizing this algorithm, we then apply a *fixed-point* analysis to study the dynamics of failure propagation and healing in the system after an

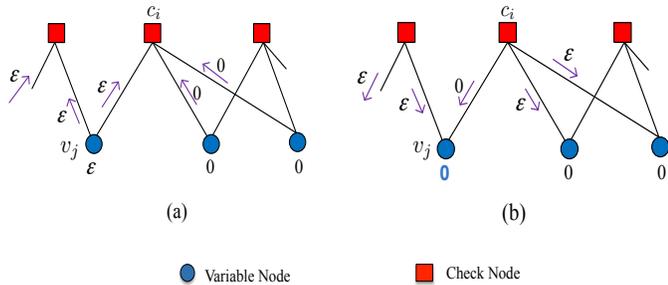


Fig. 1: One iteration of message passing in LDPC codes over a binary erasure channel.

initial disturbance.

- 3) From our fixed-point analysis, we derive a sufficient condition for choosing the network parameters to guarantee complete healing of the system. This condition provides us with simple yet critical design guidelines.

The rest of the paper is organized as follows. Section II briefly reviews the use of message passing in LDPC codes. Section III starts the analysis by applying message passing to a simple self-healing one-to-one network inspired by Buldyrev’s model. We then present the analysis for the general case of CPS in sections IV and V. Section IV describes the system model, notations, and message passing in the general CPS. Section V presents a fixed-point analysis of the proposed message passing algorithm. Extensive simulation results are shown in section VI to study the resiliency of the network for different network parameters. Section VII concludes the paper.

II. MESSAGE PASSING IN LDPC CODES

Factor graphs and message passing have been successfully employed in the analysis and design of LDPC codes [12], [13]. Let us explain the application of message passing in LDPC codes with a simple example. Fig. 1 shows part of the *Tanner graph* of an LDPC code where the circles and squares represent, respectively, *variable nodes* and *check nodes*. The variables nodes correspond to the symbols received from the channel, i.e., the channel outputs. In this particular example, we assumed a *binary erasure channel* (BEC) where the channel outputs are either received correctly or are unknown. In a BEC, there is no bit flip from 0 to 1 or vice versa. The functionality of a check node is to do a check-sum on the values of its variable nodes and making sure that they add up (in modulo 2) to zero.

The goal of message passing is to figure out the correct values of the unknown bits after multiple rounds of *message passing* between the variable and check nodes. As seen in the figure, there is one unknown bit (variable node) at the channel output. At the beginning of each iteration, every variable node v_j sends its value to all of its neighboring check nodes (Fig. 1(a)). Every check node c_i then derives what it believes about the value of each of its neighboring variable nodes, and sends it back to them as a message. To derive this value for v_j , c_i uses the messages received from all of its variable nodes

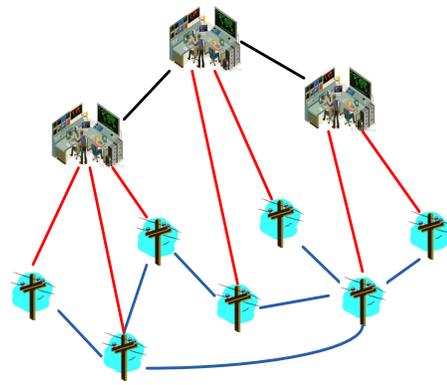


Fig. 2: An example of the graphical model of a cyber-physical system.

excluding v_j . If one or more of these messages are ϵ (erasure), then c_i cannot be of help to v_j at this round, and sends an ϵ to v_j . Otherwise, if all of these messages are either 0 or 1, c_i takes their check-sum and sends the result to v_j as what it believes v_j should be. An example of the messages sent from check nodes to variable nodes is shown in Fig. 1(b). The last two steps are repeated until the values of all variable nodes are derived, or a certain number of iterations is reached.

There are a few similarities and differences between the Tanner graph of a LDPC code and the structure of a cyber-physical system. Take the simplified model of Fig. 2 for the smart grid, where physical components of the power grid are connected to control centers or internet servers in the cyber network. Two main differences between this model and the Tanner graph of Fig. 1 could be recognized as follows:

- 1) The Tanner graph of a LDPC code is a bipartite graph in which every edge connects a check node to a variable node. In other words, there are no edges connecting the variable nodes or check nodes. In the cyber-physical system of Fig. 2, however, both of the cyber and physical systems are connected networks.
- 2) In LDPC codes, unknown (damaged) variable nodes cannot affect the functionality of the check nodes. In other words, *failure* can not propagate from variable nodes to check nodes. In cyber-physical systems, on the other hand, failure of the physical components (such as a generator) could cause a failure in the cyber network (shutting down an Internet server), and vice versa.

In this paper, we show how message passing can be applied to cyber-physical systems despite these differences.

III. MESSAGE PASSING OVER A SELF-HEALING ONE-TO-ONE MODEL

Buldyrev in [3] introduced a simple “one-to-one” model that yields an important insight into studying interdependent networks. In this model, it is assumed that two networks, say A and B, have the same number of nodes, N . The state (failed or alive) of a node in the network A directly depends on the state of the corresponding node in network B. Fig. 3 shows

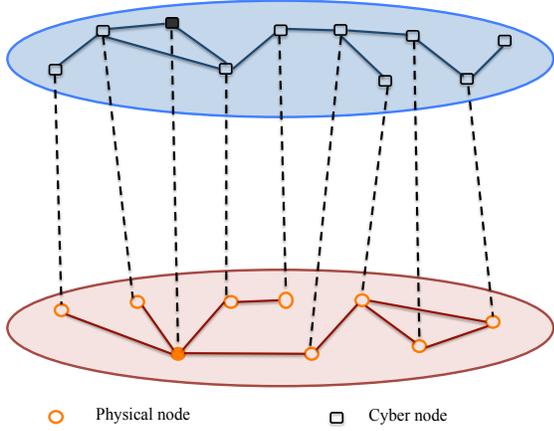


Fig. 3: A “One-to-One” interdependent model for a cyber network and a physical network.

such a one-to-one model of a cyber-physical network. There will be an initial attack on the physical network failing each physical node with a probability ϵ . Failures then propagate not through the physical network, but from the physical nodes to the cyber nodes and then through the cyber network. A cyber node with only failed cyber neighbors will fail, hence failing its underlying physical node. As time passes and failure propagates between the two networks in several *iterations*, a catastrophic cascade of failures may occur.

In Buldyrev’s model, if a physical node fails, the corresponding cyber node will also be lost, and there is no healing capability for either physical or cyber nodes. We slightly modify this model to consider a healing ability for cyber nodes. We assume that a cyber node which is not isolated from the cyber network can heal its failed physical node. That is, a cyber node with at least one healthy cyber neighbor still has access to the cyber network’s data and can heal its physical node.

We capture the propagation of failure and healing between the nodes as *defection* (D) and *healing* (H) messages exchanged between them, and apply message passing analysis tools to study the evolution of the cascade in this interdependent network. We may look at this evolution within one (any) iteration, and see how the failure probability changes for physical nodes. If this probability increases at the end of the iteration, then a cascade will occur, and if it decreases then the network will heal completely.

Let us consider the first iteration after the initial attack. Each physical node is failed at the beginning with probability ϵ . Let x_1 denote the probability of a D message from a physical node to its cyber neighbor. We will have

$$x_1 = \epsilon. \quad (1)$$

A cyber node with a failed physical node sends D messages to all its cyber neighbors telling that it has lost its physical connection. Denote the probability of this event by x_3 . Since each cyber node is connected to only one physical node, we

have

$$x_3 = x_1 = \epsilon. \quad (2)$$

A cyber node with only failed cyber neighbors sends a D message to its physical node, otherwise it sends a H message healing the physical node. If we denote the probability of the former (sending a D message to the physical node) by x_2 , it is given by

$$x_2 = \rho(x_3) = \rho(\epsilon), \quad (3)$$

where

$$\rho(x) = \sum_{i \geq 2} \rho_i x^i, \quad (4)$$

is the degree distribution of the cyber nodes and $\sum_{i \geq 2} \rho_i = 1$. Here, ρ_i is the fraction of cyber nodes with i cyber neighbors. We assumed that each cyber node is connected to at least two other cyber nodes.

Now that we are at the end of the first iteration, let us denote the probability of a physical node failure by x_0 . We have

$$x_0 = \Pr \left\{ \text{Receiving a D message from the cyber node} \right\}, \quad (5a)$$

$$\Rightarrow x_0 = x_2 = \rho(\epsilon). \quad (5b)$$

Note that for $0 \leq \epsilon < 1$, we have

$$x_0 = \rho(\epsilon) = \sum_{i \geq 2} \rho_i \epsilon^i < \sum_{i \geq 2} \rho_i \epsilon = \epsilon. \quad (6)$$

Therefore, at the end of the first iteration, the probability of failure for a physical node decreases. The same analysis as above can be carried out for any iteration j . If we denote the physical node failures at the beginning of iterations j and $j+1$ by $x_0^{(j)}$ and $x_0^{(j+1)}$ respectively, we obtain

$$x_0^{(j+1)} < x_0^{(j)}, \quad (7)$$

which means that our simple (and somewhat intuitive) healing rule for Buldyrev’s network will always lead to complete healing of the network. This of course will not be the case for more complicated network models with complex contagion and healing rules. However, the message passing approach used in this section can be generalized to develop a framework for studying such cases. The rest of this paper is dedicated to this task. Section IV sets up the network model and formulates the message passing problem for the general case. Section V then generalizes the technique used here by applying a *fixed point analysis* to study the evolution of the cascade in the network.

IV. PROBLEM FORMULATION

This section presents a factor graph framework to study message passing in cyber-physical systems. First, we explain our network model for the physical and cyber networks. We will then describe our models for the initial disturbance, healing, and contagion within each network and between the

two networks.

A. Network Model

For our analysis, we consider random networks with given degree distributions as models of cyber and physical networks. This enables us to model random networks with arbitrary degree distributions such as scale-free networks with a power law degree distribution [14], and Erdős-Rényi random graphs with a Bernoulli degree distribution [15]. We define cyber (physical) degree of a node as the number of nodes in the cyber (physical) network connected to the node. In a similar fashion to LDPC codes, we use polynomials to represent the degree distributions of the networks. We denote by

$$\rho(x) = \sum_{i \geq 2} \rho_i x^i, \quad (8)$$

and

$$\lambda(x) = \sum_{i \geq 2} \lambda_i x^i \quad (9)$$

the degree distributions of the cyber and physical networks, respectively. In the above, ρ_i is the fraction of cyber nodes with cyber degree i and λ_i is the fraction of physical nodes with physical degree i . In practical systems, each node often has more than one neighbor. Hence, we assume $i \geq 2$ in above equations.

To capture the interconnections between the two networks, two more polynomials are needed: one for physical degree distribution of cyber nodes, and one for cyber degree distribution of physical nodes. However, in order to simplify the presentation of results, we assume that each cyber node can control a physical nodes, while each physical node is connected to one cyber node. The analysis for the general case of degree distributions could be carried out along the same lines as the analysis in this paper.

B. Initial Disturbance, Contagion, and Healing

Here we explain our models for the initial disturbance, contagion within each system and between the two, and healing of the physical system by the cyber system. Our methodology, however, could be extended to a wide range of models.

- **Initial disturbance:** We adopt a simple model assuming that each physical node initially fails with a small probability ϵ , where $\epsilon \ll 1$. In this paper, we only consider the initial disturbance for the physical network. The analysis for the case of a cyber attack could be conducted in a similar fashion.
- **Contagion within physical network:** After being defected, a physical node may defect each of its neighbors with probability p . This probabilistic model is commonly used in the literature for a range of applications [16].
- **Healing of physical nodes:** A cyber node heals a physical node if that physical node is its only defected physical neighbor. An example of this could be a control center that has all the measurements but one from the power

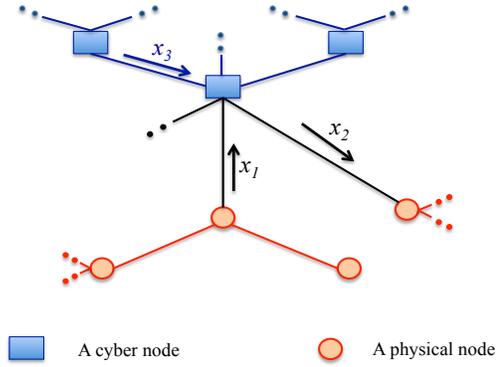


Fig. 4: An example of messages in a cyber-physical system.

grid, so it may derive the phase or voltage value for the remaining component.

- **Contagion from physical to cyber system:** A cyber node with no functioning physical neighbor will go out of service. A case for this could be an internet server who loses its power supply in a power outage.
- **Contagion within cyber system:** If all cyber neighbors of a cyber node are out of service, the cyber node itself will go out of service. An example could be an internet server whose neighboring servers have all been disconnected from the network.

C. Message Passing in Cyber-Physical Systems

In our model, the interactions between the nodes are represented by messages. Accordingly, all sorts of contagions and the healing process scenarios explained above could be interpreted in a message passing framework as follows:

1) Defection (D) message:

- A defected physical node sends a defection message D to its cyber neighbors. It also sends a message D to each of its physical neighbors with probability p .
- A defected cyber node sends a D-message to its cyber and physical neighbors.
- A functioning cyber node that cannot heal a physical node sends a D-message to that node.

2) Healing (H) message: A cyber node which is able to heal a physical node sends a healing message H to that node.

Note that these messages are introduced to only capture the interactions in the cyber-physical system, while there may not be really exchanged between the nodes in the underlying networks. However, we use x_1, x_2 and x_3 messages to analyze the interactions within a cyber-physical network. Fig. 4 shows these messages.

V. FIXED-POINT ANALYSIS OF CYBER-PHYSICAL SYSTEMS

In mathematics, a *fixed point* of a function is defined as an element in the function's domain that can be mapped to itself. That is, x is a fixed point of $f(x)$ iff $x = f(x)$.

Particularly, if $f(0) = 0$ and the equation $x = f(x)$ does not have a non-zero solution, then $x = 0$ is the only fixed point of $f(x)$. In our cyber-physical system, let $x_0 \in [0, 1]$ denote the failure probability of physical nodes. A fixed-point analysis on the system, if possible, will result in deriving a function $f(\cdot)$ such that $x_0 = f(x_0)$. Now, for a given set of the system parameters, if this equation does not have a solution in $(0, 1]$, then the system is bound to completely heal. In this section, we obtain such function $f(\cdot)$, and derive a sufficient condition for the equation above to not have a solution in $(0, 1]$. The following theorem derives the fixed-point equation $f(x_0) = x_0$.

Theorem 1. *For the cyber-physical system defined in Section IV with degree distribution pair (λ, ρ) , and parameters a and p , the fixed-point equation is obtained as follows:*

$$x_0 = f(x_0), \quad (10)$$

where

$$f(x_0) = \left(x_0 + (1 - x_0) \left(1 - \lambda(1 - px_0) \right) \right) \times \left(\left[1 - (1 - (x_0 + (1 - x_0)(1 - \lambda(1 - px_0))))^{a-1} \right] \times \left[1 - \rho((x_0 + (1 - x_0)(1 - \lambda(1 - px_0)))^a) \right] \right). \quad (11)$$

Proof: After a disturbance has occurred in the physical network, a D-message will be generated by the failed nodes. In order to obtain the fixed-point equation, we need to define x_1 , x_2 and x_3 , rigorously. According to their definitions x_1 , x_2 and x_3 could be written as follows:

$$x_1 = \Pr \left\{ \left(\text{The physical node itself has failed} \right) \cup \left(\text{The node has not failed} \cap \text{At least one of its physical neighbors has failed} \right) \right\}, \quad (12a)$$

$$\Rightarrow x_1 = x_0 + (1 - x_0) \left(1 - \lambda(1 - px_0) \right), \quad (12b)$$

$$x_2 = \Pr \left\{ \text{At least one physical neighbor has failed} \cap \text{At least one cyber neighbor is healthy} \right\}, \quad (13a)$$

$$\Rightarrow x_2 = 1 - (1 - x_1)^{a-1} (1 - \rho(x_3)), \quad (13b)$$

$$x_3 = \Pr \left\{ \text{All physical neighbors of a cyber node have failed} \right\} \quad (14a)$$

$$\Rightarrow x_3 = x_1^a. \quad (14b)$$

Also, x_0 can be defined as

$$x_0 = \Pr \left\{ \text{A physical node fails} \cap \text{Its cyber neighbors cannot heal it} \right\} \quad (15a)$$

$$\Rightarrow x_0 = x_1 \times x_2. \quad (15b)$$

We can eliminate x_3 by substituting (14b) into (13b), obtaining x_2 as a function of x_1 . Substituting (12b) into (13b) and then (15b) yields (11). ■

Remark 1: In order to derive equations (12) - (15), we need to maintain independence between the incoming messages to each cyber or physical node. Such an independence is guaranteed if the network is *cycle-free* [13]. More accurately, the fixed-point equation of (11) for x_0 holds only if the network does not contain cycles. An analysis along the same lines of Appendix A in [13] shows that such a cycle-free structure is achieved in our model as the network becomes larger.

A. A Sufficient Condition for Healing

Once the fixed-point equation is derived for a given set of contagion and healing rules, it can be utilized in many ways to gain useful insights into the network design. The following theorem, for example, employs equation (11) to obtain a sufficient condition for the system to heal completely.

Theorem 2. *The cyber-physical system described in Section IV with degree distribution pair (λ, ρ) , and parameters a and p heals if*

$$x_0 < \frac{1}{(a-1)(1+p\lambda'(1))^2} \quad (16)$$

Proof: By taking Taylor series from the right side of (10) at $x_0 = 0$, we obtain

$$x_0 = (a-1)(1+p\lambda'(1))^2 x_0^2 - 0.5(a-1)(1+p\lambda'(1)) \times \left[(a-2)(1+p\lambda'(1)) + 2p(2\lambda'(1) + p\lambda''(1)) \right] x_0^3 + O(x_0^4). \quad (17)$$

For the fixed-point equation not to have a solution in $(0, 1]$, it suffices to have $x_0 > f(x_0)$ in this interval. For this to hold when x_0 is small ($x_0 \ll 1$), it is enough to show that x_0 is larger than the first term in the right side of (17). That is to have

$$x_0 > (a-1)(1+p\lambda'(1))^2 x_0^2, \quad (18)$$

which leads to inequality (16). ■

Theorem 2 provides some interesting intuitions. First, note that

$$\lambda'(1) = \sum_{i \geq 2} i \lambda_i \times x^{i-1} |_{x=1} = \sum_{i \geq 2} i \lambda_i, \quad (19)$$

is the average degree of the physical nodes. Theorem 2 indicates the necessity of a low average degree for the physical nodes for achieving a resilient system. This is because in our model, physical nodes with higher degrees can defect more nodes. Second, this theorem suggests that the number of physical nodes under each cyber node, a , should be kept

TABLE I: ϵ_{th} and ϵ_{max} for different network parameters and severity of the initial disturbance.

Table I.A: ϵ_{th} and ϵ_{max} for variation of a

p	a	$\lambda(x)$	$\rho(x)$	ϵ_{th}	ϵ_{Max}
0.8	3	x^2	x^3	0.0740	0.1002
	5			0.0369	0.0482
	8			0.0211	0.0271

Table I.B: ϵ_{th} and ϵ_{max} for variation of p

p	a	$\lambda(x)$	$\rho(x)$	ϵ_{th}	ϵ_{Max}
0.4	4	x^2	x^3	0.1028	0.1621
0.6				0.0688	0.0973
0.8				0.0493	0.0650

Table I.C: ϵ_{th} and ϵ_{max} for variation of $\lambda(x)$

p	a	$\lambda(x)$	$\rho(x)$	ϵ_{th}	ϵ_{Max}
0.5	3	x^2	x^3	0.1250	0.1933
		x^5		0.0408	0.0525
		x^8		0.0200	0.0250

small. This increases the chance of healing physical nodes since a cyber node needs to have all but one measurements to heal a physical node. Finally, the theorem implies that smaller values of p are desirable, which is expected.

It is noteworthy that the application of our proposed message passing framework is not limited to the particular setting explained in section IV. This framework can be applied to any set of contagion models, healing rules, and network structures for which a fixed-point analysis can be carried out. In the next section, simulation results are shown to investigate the effect of network parameters on the resiliency of the cyber-physical system.

VI. NUMERICAL RESULTS

In this section, we numerically simulated the cyber-physical system defined in section IV for different network parameters. we fixed a , λ , and p , and found ϵ_{th} and ϵ_{max} . Here, ϵ_{th} is the largest initial disturbance satisfying the sufficient condition of Theorem 2, and ϵ_{max} is the largest initial disturbance that can be tolerated by the network. The following observations can be made from Table I:

- The results in Table I.A indicate that increasing a reduces the values of ϵ_{th} and ϵ_{max} . In fact, a large a increases the chance of receiving erasures by a cyber node from its physical neighbors. This reduces the resiliency of the system.
- The results in Table I.B confirm that reducing p leads to less vulnerability of physical nodes from their physical neighbors.
- Table I.C shows that a less connected physical network results in larger values of ϵ_{th} and ϵ_{max} and, hence, a higher resiliency.

VII. CONCLUSION

We introduced a factor graph representation of cyber-physical systems and applied message passing to investigate resiliency of inter-dependent cyber-physical systems. We provided a fixed-point analysis to study the evolution of the system in the presence of both self-healing and propagation of failures. Our analysis resulted in a sufficient condition on choosing the network parameters for the system to completely heal after an initial disturbance. In addition, we used the fixed-point equation to numerically obtain the most severe disturbance that can be tolerated by the network. We also provided simulation results to demonstrate how the ability of self-healing improves network resiliency against the failures.

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