Design and performance of a miniature free piston expander

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Heat engines are known for high power and energy densities compared to chemical batteries. The availability of free thermal energy—solar and ‘waste heat’ from electronics provide an opportunity for power generation at miniature length scales (millimeter). Though free piston based expanders at macro length scales (centimeter and above) are found to be suitable for heat energy harvesting, their implementation at miniature scales are challenged by significant parasitic losses such as heat loss and pumping work, yielding low thermal efficiencies. In this manuscript, through physics-based models the behavior and performance of a miniature free piston expander that operates on an open cycle was investigated. Here, the design space of the free piston expander with an objective to achieve an efficiency of at least 15% was explored. Three observations are reported that contribute to higher efficiency operation: (1) a higher injection pressure; (2) an optimum nondimensionalized duration of injection time of 1.5; and (3) softer springs, lower loads, and heavier pistons. A sample calculation showed that a centimeter-sized expander can generate an output power of 2.24 W at 18% efficiency. This study shows that both the performance parameters of the expander, namely efficiency and output power are sensitive to injection pressure of the working fluid compared to the time duration of the working fluid injection. The study reveals that the miniature free piston expander is promising for low temperature waste-heat harvesting.

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1. Introduction

Power generation via miniature thermal energy harvesters, \(\Delta O(10^{-2})\) m, has been a topic of interest due to many foreseeable applications such as powering sensors meant for environmental (air quality, noise pollution) and structural (bridges, high-rise buildings, and stadiums) health monitoring that require a sustained, reliable milli-Watt power source [1]. Despite other existing technologies such as rechargeable batteries, photovoltaic cells and vibration energy harvesters, thermal harvesters are a superior due to their high energy and power densities [2]. An example thermal harvester is a phase-change based harvester which offer the advantage of energy source flexibility, that is, it can operate from different external heat sources [3]. A typical phase-change based harvester setup comprises of four components: a boiler to scavenge available energy and generate pressurized steam, an expander to convert heat energy of the pressurized steam into mechanical work, a condenser to change the state of expended steam into liquid, and a pump to circulate the condensed liquid into the boiler.

Typically, low temperature harvesters are based on an Organic Rankine Cycle (ORC) [4] which employs refrigerant (e.g. HCFC-123 [5]) as a working fluid in traditional high-speed turbines [6] and crankshaft-based piston expanders [7], and less popular screw [8], scroll expanders [9], and Tesla turbines [10]. To improve the performance of the ORC systems, researchers analyzed different working fluids such as HCFC-123 [5] and HFOs [11] against traditional fluids (e.g. water). Liu et al. studied the effect of different working fluids on ORC for waste heat recovery application [12]. Sprouse III and Depcik have reviewed recent trends in ORC systems meant for harvesting waste heat from internal combustion engines and showed that the fuel economy can be improved by up to 10% with the use of modern refrigerants and advancements in expander technology [13]. Most ORC-based harvesters are macroscale systems which benefit from scaling laws that offer high manufacturing tolerances, less heat loss, and less pumping work. Unfortunately, as the size of these thermal systems decrease, the ratio of friction, leakage, and heat losses to input energy become significant in the
expander units, resulting in poor thermal efficiencies [14]. A method to improve thermal efficiency is by eliminating the crankshaft assembly and associated friction losses; this can be accomplished by implementing a free-piston architecture of the expander which also offer other advantages — simpler construction and lower fabrication costs.

Most prior research has focused on the design of macroscale free-piston expanders (FPE) with a target output power in the 20—100 W range [15,16]. Here, researchers found that FPEs are suitable for heat energy harvesting application. The FPE implementation at miniature scales are not completely studied; however, theoretically, FPE utilization at miniature length scales is challenged by significant parasitic losses such as heat loss and pumping loss, as aforementioned. On the contrary, the scaling laws benefit FPEs in terms of specific output power [14]. To evaluate the suitability of FPE at miniature scales and to optimize FPEs, one needs to identify their important design and operating parameters. Prior efforts recorded in literature have made progress on design and operation of thermal harvesters on several fronts — different working fluids (HCFC-123 [5], HFOs [11], and air [15]) have been considered — different boiler [17] and superheater [18] designs for miniature length scales have been proposed — new lumped-parameter-based [19] and numerical-based models [16] of the free-piston expander (FPE) have been developed. Many efforts are specific to low temperature waste-heat harvesting [20] and multiple have resulted in real prototypes across different length scales ranging from mesoscale [20] to macroscale [16,21]. More recent prototype development has included low grade energy use in ORC-style free piston expanders with real devices characterized by several groups [15,16,22]. In these works, a linear electric generator was also incorporated for real-world demonstration of power output. These efforts have produced devices with bore diameters up to 80 mm [16] that can generate an output power of up to 96 W [22].

This present work builds on this growing interest through exploration of free piston expander design space on the smaller scale, specifically in an ORC-style operation. There is significant opportunity to identify physical and operating parameters in this development space. This includes dynamics and thermodynamic modeling framework that provides insight into the critical parameters and means to optimize FPE performance. This work builds on our own prior, physics-based model [19]. Like other works [23], the initial effort modeled the FPE as a closed cycle operation, where the steam injection and exhaust processes were modeled as heat addition and rejection processes respectively. In this work, the FPE is modeled close to its final real-world operation, that is as an open cycle more similar to the larger scale prototype expanders (Fig. 1).

This work specifically investigates the behavior, performance, and explores the design space of a centimeter-sized FPE modeled to operate on an open cycle (Fig. 1). In this cycle, a hot, pressurized working fluid (air) is injected into the FPE cavity through a valve to produce PdV work at the piston end (Fig. 2). The model is based on a lumped parameter approach developed using first principles. The following sections describe the FPE design, a physics-based model of the FPE, and results that describe FPE behavior. Results show that up to 15% efficiency can be achieved when supplied with hot, compressed air at 373 K temperature and 250 kPa pressure.

2. FPE design

The FPE design is similar to a traditional steam-expander system, where hot, pressurized working fluid (steam or vapor) is injected into the FPE cavity ‘CV’ through a valve to produce PdV work by the linear motion $x(t)$ of the piston $m$ (Fig. 2).

The FPE operates on an open cycle comprised of injection (1 $\rightarrow$ 2), expansion (3 $\rightarrow$ 4), and exhaust (4 $\rightarrow$ 5) processes as depicted in Fig. 1. The cycle begins with the piston at state 1 ‘Top Dead Center’ (TDC) where hot, high pressure working fluid is injected into the CV by the opening of the valve. This results in pressurization of the CV and moving of the piston towards ‘Bottom Dead Center’ (BDC). The valve remains open until the piston reaches state 3. Note that the first phase of injection process 1 $\rightarrow$ 2 is pressure driven — pressure difference across the valve, while the second phase of the injection process 2 $\rightarrow$ 3 is a timed injection for a predetermined time duration $t_{23}$. Then, the valve closes, the expansion process 3 $\rightarrow$ 4 begins where the CV expands until state 4. Next, the valve opens and blowdown phase of the exhaust process begins where the expended working fluid is discharged out of the CV instantaneously and adiabatically to ambient pressure $P_0$ (4 $\rightarrow$ 5). Finally, the displacement phase of the exhaust process begins (5 $\rightarrow$ 1) where the piston travels from BDC to TDC scavenging the remaining working fluid from the CV. This completes one operating cycle of the FPE. Note the injection (IR) and expansion (ER) ratios are defined as $IR = \frac{V_1}{V_2}$ and $ER = \frac{V_3}{V_4}$.

It is worth noting that, unlike standard reciprocating engines where TDC and BDC are geometrically determined, the TDC and BDC in the FPE are determined by the operating conditions.

3. Model

A physics-based model of the FPE is developed using the lumped-parameter approach to investigate the FPE behavior,
A lumped parameter model of the FPE shown in Fig. 2 is derived by applying Newton’s second law for the piston mass of the working fluid, and a linear mass flow-rate equation for the valve. Mathematical statements of these principles are:

\[
\Delta T = \frac{1}{\text{Mc}_T \omega} \left[ M_T (R + c_p \Delta T_1 - c_v \Delta T) - hT_o \Delta T - M_e R T_o (1 + \Delta T) \right] - P_o V_o \omega (1 + \Delta P) \Delta V
\]

\[
\Delta P = \frac{1}{P_o V_o \omega (1 + \Delta V)} \left[ (M_1 - M_e) R T_o (1 + \Delta T) + M R T_o \omega \Delta T - P_o V_o \omega (1 + \Delta P) \Delta V \right]
\]

Here, it is noteworthy to mention that Eq. (10) is obtained by taking a first derivative of ideal gas law Eq. (4) with respect to the nondimensional time \( \bar{t} \).

To generate a pressure-volume diagram, the model Eqs. (7)–(10) is numerically integrated in the order 1 \( \rightarrow \) 2 \( \rightarrow \) 3 \( \rightarrow \) 4 \( \rightarrow \) 5 \( \rightarrow \) 1 (Fig. 1) starting with state 1: \( \Delta V_1, \Delta P_1, \Delta T_1 \), \( \Delta V_1 = 0 \). A proper choice of state variables at state 1: \( \Delta V_1, \Delta P_1, \Delta T_1 \) are necessary to obtain a steady state solution (see Appendix for details). The resulting volume \( \Delta V_2 \), pressure \( \Delta P_2 \), and temperature \( \Delta T_2 \) from the first phase of the injection process, 1 \( \rightarrow \) 2 is computed by numerically integrating Eqs. (7)–(10) from the initial condition \( \Delta V_1, \Delta P_1, \Delta T_1 \), \( \Delta V_1 = 0 \) for a time \( \bar{t}_{12} \) such that the CV pressure \( P_2 \) equals injection pressure \( P_2 = P_o \). The resulting volume \( \Delta V_3 \), pressure \( \Delta P_3 \), and temperature \( \Delta T_3 \) during the second phase (timed injection) of the injection process, 2 \( \rightarrow \) 3 is computed by numerically integrating Eqs. (7)–(10) from the initial condition \( \Delta V_2, \Delta P_2, \Delta T_2 \) \( \Delta V_2, \Delta P_2, \Delta T_2 \) for a predetermined time duration \( \bar{t}_{23} \). The volumes, pressures, and temperatures \( \Delta V_3, \Delta P_3, \Delta T_3 \) and \( \Delta V_3, \Delta P_3, \Delta T_3 \) from the expansion and exhaust (displacement phase) processes 3 \( \rightarrow \) 4 and 5 \( \rightarrow \) 1 are computed by numerically integrating Eqs. (7)–(10) from the initial conditions \( \Delta V_3, \Delta P_3, \Delta T_3 \) and \( \Delta V_3, \Delta P_3, \Delta T_3 \) for times \( \bar{t}_{34} \) and \( \bar{t}_{51} \), respectively such that the piston velocity

\[
\frac{d\Delta V}{dt} = \Delta P
\]

\[
\frac{d\Delta V}{dt} = \frac{b}{m \omega} \Delta V - \frac{k}{m \omega^2} \Delta V
\]
\[ \Delta V_i = \Delta V_f = 0. \] It is worth noting that the times \( \tau_{12}, \tau_{23}, \tau_{51} \) are ‘a priori’ and is determined during the integration. It is assumed that the blowdown process \( 4 \to 5 \) is instantaneous, isentropic, and occurs at zero piston velocity \( (\Delta V_i = \Delta V_f = 0) \) and ambient pressure such that \( \Delta P_f = 0 \). Therefore, the temperature of the working fluid at the end of the exhaust (blowdown phase) process \( 4 \to 5 \), \( \Delta T_5 \) is computed using the adiabatic relation given by Eq. (11).

\[ \Delta T_5 = (1 + \Delta T_4)\left(\frac{1 + \frac{\Delta P_d}{P_0}}{1 + \frac{\Delta P_f}{P_0}}\right)^{\frac{1}{\gamma}} - 1 \]  

4. Results and discussion

The centimeter-sized FPE shown in Fig. 2 modeled with \( m = 0.034 \text{ kg}, \) \( k = 1000 \text{ N/m}, \) \( b = 10 \text{ N-s/m}, \) \( h = 0 \text{ W/K}, \) \( \beta = 0.0064 \text{ kg/Pa-s}, \) and \( V_o = 0.785 \text{ cm}^3 \) (1 cm in diameter and 1 cm of nominal length) is treated as the reference case. The numerical values of some of these FPE parameters are based on the prior work by the authors [20,26]. Standard temperature and pressure conditions of \( T_o = 298 \text{ K} \) and \( P_o = 101 \text{ kPa} \) are chosen for the ambient state. The working fluid is a hot compressed air, assumed to behave like an ideal gas above the injection temperature \( (T_i > 23 \text{ K}) \). This allows formal concentration on the FPE operating cycle. Future efforts and experiment will consider phase change effects of working fluids already under investigation through experimental boiler development [18]. These are generally ‘dry’ fluids with a negative slope to the saturation curve. This allows expansion of the fluid within the FPE while maintaining a superheat condition.

4.1. FPE behavior and performance

A representative behavior and performance of the FPE with the working fluid injection temperature \( T_i = 373 \text{ K} \), pressure \( P_i = 250 \text{ kPa} \), and duration \( \tau_{23} = 1.5 \) for the reference case is shown in Fig. 3. The pressure-volume diagram for one operating cycle \( 1 \to 2 \to 3 \to 4 \to 5 \to 1 \) shows that the first phase of the injection process, process \( 1 \to 2 \) occurs at near constant volume \( (V_{1 \to 2} = 0.781 \text{ cm}^3) \) and the second phase of the injection process, process \( 2 \to 3 \) occurs at near constant pressure \( (\approx 250 \text{ kPa}) \) that results in the volume increase of the CV, \( V_3 = 1.212 \text{ cm}^3 \) (Fig. 3a). Following the injection process \( 1 \to 2 \to 3 \), the FPE expands isentropically during the process \( 3 \to 4 \to 5 \) to about \( V_4 = 1.461 \text{ cm}^3 \), where the pressure is above atmosphere \( (P_4 = 190 \text{ kPa}) \). Next, the blowdown phase of the exhaust process \( 4 \to 5 \) occurs, where the expended working fluid is discharged due to the pressure gradient across the CV and exhaust system. Finally, the FPE undergoes the displacement phase of the exhaust process \( 5 \to 1 \), where the residual working fluid is scavenged from the CV until the volume of the CV becomes \( V_1 = 0.781 \text{ cm}^3 \). The ratio of the numerical integration of PV diagram (PdV work) and the enthalpy added to the FPE \((M_2C_{p2} \Delta T_2 - M_1C_{p1} \Delta T_1)\) over one operating cycle gives the FPE energy conversion efficiency \( \eta = 13.78\% \). Here, the injection ratio is \( IR = 1.55 \) and the expansion ratio is \( ER = 1.2 \). A corresponding plot of the mass of the working fluid in the CV for an operating cycle is shown in Fig. 3b.

The peak internal cavity temperature across an operating cycle occurs at thermodynamic state \( 2, T_2 = 403.7 \text{ K} \) (Fig. 3c), which is above the injection temperature \( (T_i = 373 \text{ K}) \) – a result of the enthalpy addition to the cavity (CV). Therefore, the design temperature of the FPE should be above the working fluid injection temperature. In an operating cycle, the time durations and velocity amplitudes for the different processes are unequal (Fig. 3d). For instance, the forward motion of the piston \( (1 \to 5) \) that comprises of multiple processes lasts for a time duration \( \tau_{15} = 17.6 \text{ ms} \) while the backward motion of the piston \( (5 \to 1) \), lasts longer with a time duration \( \tau_{51} = 36.8 \text{ ms} \). It is worth noting that this return motion can be governed by careful FPE design. For example, a motion controlled by an opposing piston expansion will adopt a faster return to TDC. This is discussed in greater detail in following sections.

4.2. Effect of operating conditions

The effect of operating conditions, namely working fluid injection temperature \( T_i \), pressure \( P_i \), and time duration \( \tau_{23} \) on the performance of the FPE characterized in terms of efficiency \( \eta \) is
presented in Fig. 4. Here, one operating condition (e.g. \( T_i \)) is varied while the other two operating conditions \((P_i, T_23)\) are held constant. For each set of operating conditions, the model Eqs. \( (1)-(5) \) are solved simultaneously to first generate a PV diagram, thereupon based on the PV diagram, the corresponding efficiency \( \eta \), IR, and ER are computed.

The model Eqs. \( (1)-(5) \) predicts that for fixed \( P_i \) and \( T_23 \), an increase in \( T_i \) does not affect the efficiency \( \eta \) (Fig. 4a). A closer look at the PdV work and enthalpy added shows that they remain constant for any \( T_i \). Note that the efficiency which depends on IR and ER is also found to be constant. On the contrary, both the IR and ER increase with \( P_i \) for fixed \( T_i \) and \( T_23 \), resulting in an increase in the efficiency \( \eta \) (Fig. 4b and c). It is evident from the figure that the \( \eta \) is more dependent on IR than ER. As the expander is “free-piston” type, the stroke length or BDC volume is not more dependent on IR than ER. As the expander is

4.3. Significant operating parameter

Among the three operating parameters, only \( P_i \) and \( T_23 \) are found to dominate the performance of an FPE. To establish the most significant parameter among the two, a sensitivity analysis has been performed on the FPE outputs —efficiency \( \eta \) and output power \( \Omega \)—by defining sensitivities \( \frac{\delta \eta}{\delta P_i}, \frac{\delta \eta}{\delta T_23}, \frac{\delta \Omega}{\delta P_i}, \) and \( \frac{\delta \Omega}{\delta T_23} \), where \( \delta \) denotes partial difference operator. For the analysis, a parametric sweep is performed on the model Eqs. \( (1)-(5) \) with FPE inputs \( P_i \) in the range \([1.5, 3.5]\) and \( T_23 \in [0.15, 1.5, 5, 10] \), and sensitivities are computed and plotted (Fig. 5a and b). Here, the output power is
nondimensionalized with the scaling parameter \( P_1, V_{2a} \). For a specific condition of \( P_1 \) and \( T_{23} \), if \( \frac{\partial^2}{\partial t^2} > \frac{m}{k} \), then the efficiency \( \eta \) of the FPE is sensitive to injection pressure \( P_1 \) compared to injection time duration \( T_{23} \) and vice versa. This sensitivity analysis showed that the data points (sensitivities) depicted by \( □, ■, ○, △, \) and \( ▼ \) are above the line of equal sensitivity, indicating that the FPE performance parameters \( \eta \) and \( \Omega \) are more sensitive to \( P_1 \) than \( T_{23} \) (Fig. 5a and b).

### 4.4. Effect of physical parameters

The FPE physical parameters, namely spring stiffness \( k \), load \( b \), and piston mass \( m \) affect its efficiency \( \eta \) and operating frequency \( f \) (Fig. 6a–c). Here, one parameter (e.g. \( k \)) is varied while the other two parameters \((m, b)\) are held constant.

An increase in spring stiffness \( k \) restrains the piston motion causing shorter displacement strokes \((V_2 \rightarrow V_4)\), hence a lower PdV work and a decrease in the efficiency \( \eta \). Also, the shorter displacement strokes reduce the piston-travel time, and hence an increase in the operating frequency \( f \) (Fig. 6a). It is worth noting the significance of \( k \) in this instance: this fundamentally represents the reset mechanism for the FPE. Careful design of \( k \) can result in both the best-case operating frequency and return to TDC functionality. Further, in a double-acting FPE design, the action of the opposing piston to limit the effective \( k \) would be critical.

An increase in load \( b \) resists the piston motion causing shorter displacement strokes \((V_2 \rightarrow V_4)\), hence a lower PdV work and a decrease in the efficiency \( \eta \). Despite the displacement stroke is shorter, the resistive behavior of the load \( b \) slows the piston velocity resulting in longer cycle times or lower frequencies \( f \) (Fig. 6b). It is worth noting, the ER and IR are found to decrease with the increase in \( b \) —consistent with efficiency trends.

Unlike stiffness \( k \) and load \( b \), an increase in piston mass \( m \) results in increasing the efficiency due to higher IR and ER (Fig. 6c). For instance, IR = 1.56 and ER = 1.2 for \( m = 0.034 \) kg, and IR = 3.9 and ER = 1.5 for \( m = 0.34 \) kg. The higher ER and IR implies larger displacement strokes, and is a result of higher momentum gained by the piston during the injection process 1 → 3. The authors acknowledge that the FPE operates close to its resonant frequency, and an increase in piston mass \( m \) results in decreasing the frequency \( f \) given by the relation \( f = \sqrt{k/m} \).

To achieve an efficiency of at least 15%, based on the physical parameters from Fig. 6a–c, an FPE with \( k = 0.7 \) kN/m, \( b = 7 \) N-s/m, and \( m = 0.068 \) kg is studied (Fig. 6d). The model predicts that the FPE generates a PdV work of 140 mJ at an efficiency of 18%, which is operating at 16 Hz frequency. This corresponds to an output power of 2.24 W. The achieved operating efficiency and output power places the FPE well above many comparably sized phase-change, low-temperature systems disclosed in literature [27].

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**Fig. 6.** Effect of FPE physical parameters: spring stiffness \( k \), load \( b \), and piston mass \( m \), on efficiency \( \eta \) and frequency \( f \) for the reference case with \( P_1 = 250 \) kPa, \( T_1 = 373 \) K, and \( T_{23} = 15 \). (a) Fixed \( b = 10 \) N-s/m and \( m = 0.034 \) kg at different \( k \in [100, 1500] \) N/m. (b) Fixed \( k = 1000 \) N/m and \( m = 0.034 \) kg at different \( b \in [2, 15] \) N-s/m. (c) Fixed \( b = 10 \) N-s/m and \( k = 1000 \) N/m at different \( m \in [0.034, 0.34] \) kg. (d) PV diagram of the FPE with \( k = 700 \) N/m, \( b = 7 \) N-s/m, and \( m = 0.068 \) kg.

**Fig. 7.** Effect of (a) heat loss coefficient \( h \) and (b) mass flow rate coefficient \( \beta \) on the efficiency of an FPE designed with physical parameters: piston mass \( m = 0.068 \) kg, spring stiffness \( k = 700 \) N/m, and load \( b = 7 \) N-s/m operating with \( P_1 = 250 \) kPa, \( T_1 = 373 \) K, and \( T_{23} = 1.5 \).
4.5. Effect of heat and pumping losses

Using the FPE with parameters $k = 0.7\, \text{kN/m}$, $b = 7\, \text{N-s/m}$, and $m = 0.068$, the effect of heat loss and pumping loss are separately investigated over a range up to three orders of magnitude, $O(10^3)$ (Fig. 7) by choosing $\beta = 0.0064\, \text{kg/(Pa-s)}$ and $h = 0\, \text{W/K}$, respectively.

With the increase in $h$, the efficiency of the FPE decreases due to the increase in heat loss (Fig. 7a). The heat loss coefficient $h$ used in this study is in the range of $0.0025–0.25\, \text{W/K}$, three orders of magnitude; the range of $h$ is based on our previous work using model and experiment [3]. A heat loss coefficient $h$, $O(10^{-3})$ results in a heat loss that is $O(\text{PdV})$ work. Therefore, for realizing a miniature FPE, the $h$ should be limited to $-0.001\, \text{W/K}$, which results in an FPE efficiency of 18%. Note that the heat loss has no significant effect on the frequency (Fig. 7a).

An increase in mass flow rate coefficient $\beta$ increases both the efficiency and frequency, because a higher $\beta$ implies a smaller resistance to flow or lower pumping loss (pumping work) during the exhaust process $5 \rightarrow 1$ —resulting higher $\text{PdV}$ work (Fig. 7b). The pumping loss is parasitic (friction-like) in nature, modeled as a mechanical damper, and exhibits the characteristics of load $b$. Therefore, an increase in $\beta$ that is decrease in load $b$ causes an increase in frequency —consistent with trends in Fig. 6b.

5. Conclusions

This work describes the behavior, performance and explores the design space of a centimeter-sized free piston expander, which is known to have several advantages over traditional turbine-style approaches. The FPE operates as an open cycle and is modeled using first principles. Three observations are reported that help achieve higher efficiencies: (1) a higher injection pressure; (2) an optimum nondimensionalized duration of injection time of 1.5; (3) softer springs, lower loads, heavier pistons are desirable. The FPE’s physical parameters, operating conditions, and acceptable heat loss and mass flow rate coefficients which yield an efficiency of 15% and an output work on the order of a few Watts were identified. FPE performance reflects the influence of two critical parameters, injection pressure of the working fluid and duration of injection. These significantly affect output power and operating efficiency.

This study indicates the promise of the small-scale FPE approach for low temperature energy scavenging. The 15% operating efficiency is a significant improvement over many low temperature phase-change systems in literature. Further, the architecture of the FPE provides a more reliable, production ready system that can be achieved without the mechanical challenges faced by microscale turbine-based devices. It is worth noting that there are alternative turbine designs such as Tesla turbine that are less complex, and may benefit from scaling laws and produce acceptable output power at miniature length scales. The simplistic designs and fewer expander components can be especially valuable when considering modern manufacturing processes like additive manufacturing to realize thermal energy harvesters at miniature length scales.

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Appendix

Fig. A1. Transient pressure-volume diagram of the FPE in an open cycle operation. The superscripts “$'$” and “$''$” denote transient states. Note that the cycle is not closed that is states $1'$ and $1''$ are not equal. However, the FPE reaches a steady state operation (states $1'$ and $1''$ are equal) in the subsequent cycles.

An example integration illustrating the first cycle in a transient dwell-up starting from state 1 is shown in Fig. A1. To generate a pressure-volume diagram, the model Eqs. (7)–(10) is numerically integrated in the order $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 1$ starting with state $1$: $\Delta V_1$, $\Delta P_1$, $\Delta T_1$, $\Delta F_1 = 0$ as described in the Model section. But, since the system undergoes a transient phase denoted with the superscript “$'$”, the cycle is not closed where $\Delta V_1 \neq \Delta V_1'$, $\Delta P_1 \neq \Delta P_1'$, and $\Delta T_1 \neq \Delta T_1'$. However, if the process is repeated, the cycle eventually reaches steady state, the cycle closes when $\Delta V_1 = \Delta V_1''$, $\Delta P_1 = \Delta P_1''$, and $\Delta T_1 = \Delta T_1''$ —and the steady states are denoted without superscript “$'$” (solid line in Fig. A2). The integration stops when the equality has been satisfied to a specified tolerance. In practice, a function-minimization computational procedure was used, which determine the proper choice of $\Delta V_1$, $\Delta P_1$, $\Delta T_1$ that would result in a steady-state periodic cycle.

Nomenclature

$L$ Cavity nominal length (cm)
$S$ Cavity cross-sectional area (cm$^2$)
Volume ($m^3$) $V$

Mass ($kg$) $m$

Spring stiffness ($N/m$) $k$

Load ($N/s/m$) $b$

Displacement at time $t$ ($cm$) $x(t)$

Heat loss coefficient ($W/K$) $h$

Mass flow rate coefficient ($kg/(Pa-s)$) $\beta$

Mass of the working fluid ($kg$) $M$

Constant volume heat capacity of working fluid ($J/kg-K$) $c_v$

Constant pressure heat capacity of working fluid ($J/kg-K$) $c_p$

Ratio of specific heats of working fluid $\gamma$

Mass-specific gas constant of working fluid ($J/kg-K$) $R$

Temperature ($K$) $T$

Pressure ($Pa$) $P$

Temperature ($K$) $t$

Injection temperature of working fluid ($K$) $T_i$

Injection pressure of working fluid (kPa) $P_i$

Time duration for the first phase of the injection process $t_{12}$

Time duration for the second phase of the injection process $t_{23}$

Time duration for the expansion process $t_{34}$

Time duration for the forward motion of the piston $t_{15}$

Time duration for the displacement phase of the exhaust process or the backward motion of the piston $t_{51}$

Efficiency $\eta$

Frequency $f$

Abbreviations

FPE Free piston expander

ORC Organic Rankine cycle

HCCI Homogeneous charge compression ignition

TDC Top dead center

BDC Bottom dead center

CV Cavity or control volume

IR Injection ratio

ER Expansion ratio

Other Subscripts and Superscripts

FPE nominal cavity volume ($m^3$)

Numerical subscript FPE's thermodynamic state

Overbar Nondimensional term

Overdot Differentiation with respect to time

Subscript $i$ Injection condition

Subscript $o$ Ambient condition

Superscripts $'$ and $"$ Transient condition

References


