Dr. Jeanne Clelland
University of Colorado at Boulder

“Isometric embedding via strongly symmetric positive systems”

Abstract: (Joint work with Gui-Qiang Chen, Marshall Slemrod, Dehua Wang, and Deane Yang)
In this talk, I will give an outline of our new proof for the local existence of a smooth isometric embedding of a smooth 3-dimensional Riemannian manifold with nonzero Riemannian curvature tensor into 6-dimensional Euclidean space. Our proof avoids the sophisticated microlocal analysis used in earlier proofs by Bryant-Griffiths-Yang and Nakamura-Maeda; instead, it is based on a new local existence theorem for a class of nonlinear, first-order PDE systems that we call "strongly symmetric positive." These are a subclass of the symmetric positive systems, which were introduced by Friedrichs in order to study certain PDE systems that do not fall under one of the standard types (elliptic, hyperbolic, and parabolic).

As in earlier proofs, we construct solutions via the Nash-Moser implicit function theorem, which requires showing that the linearization of the isometric embedding PDE system near an approximate embedding has a smooth solution that satisfies "smooth tame estimates." We accomplish this in two steps:
(1) Show that the approximate embedding can be chosen so that the reduced linearized system becomes strongly symmetric positive after a carefully chosen change of variables.
(2) Show that any such system has local solutions that satisfy smooth tame estimates.

The main advantage of our approach is that step (2) is much more straightforward than similar results for other classes of PDE systems used in prior proofs, while step (1) requires only linear algebra.

The talk will focus on the main ideas of the proof; technical details will be kept to a minimum.

Friday, March 9, 2018
3:00 PM in 372 Jabara Hall

Please come join us for refreshments before the lecture at 2:30 p.m. in room 353 Jabara Hall.