Comparison of viscoelastic/viscoplastic models for describing the creep and ratcheting behaviors of adhesives

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Introduction

• Project
  • Durability of Bonded Aerospace Structures

• Principal Investigators & Students
  • Lloyd Smith
  • Yi Chen

• FAA Technical Monitor
  • Ahmet Oztekin

• FAA Sponsors
  • Larry Ilcewicz, Cindy Ashforth

• Industry Partnerships/Other Collaborations
  • Boeing, Will Grace, Kay Blohowiak, Ashley Tracey

• Source of matching contribution for the current award
  • WSU and Boeing
Outline

• Background
• Experiment Introduction
• Model Introduction
• Model Comparison
• Summary
Background

- Adhesive films in bonded joints
  - High shear load
  - Behave differently from adhesive bulks
  - Edge effect, higher shear near edge
  - Reliable strain measurement method

- Strain response to cyclic loads
  - Ratcheting
  - Time-dependent, Plastic
  - Material damage & fatigue life

- Constitutive model
  - Viscoelasticity, Viscoplasticity
  - Damage
Aim

- Finite Element Analysis
  - Predictive capacity
  - Parameter calibration
  - Numerical implementation

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<tr>
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Outline

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• Model Introduction

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Experiment Setup

- Toughened adhesive: EA9696 (Henkel)
- 45-degree rectangular stacked rosette strain gage
- Scarf Joint: 10°

\[
\begin{align*}
\varepsilon_1' &= \varepsilon_1 a / t \\
\varepsilon_2' &= \varepsilon_2 \cos(45^\circ) / t \\
\varepsilon_3' &= \varepsilon_3 b / t \\
\gamma_{xy} &= 2\varepsilon_2' - \varepsilon_1' - \varepsilon_3'
\end{align*}
\]

Creep & Ratcheting

- Creep input

- Ratchet input, \( R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = 0.1 \)
Fully Reversed Cyclic

- $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = -1$

- Ratcheting in tension was more significant than observed in compression

- More residual strain than observed during tensile cyclic tests

- Modulus degradation
Outline

- Background
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- **Model Introduction**
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## Model Introduction

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Linear Viscoelastic

- Time-Domain Viscoelasticity (L-VE)

\[ \tau(t) = G_0 \gamma(t) + G_0 \int_0^t g_R(t - s)\dot{\gamma}(s)ds \]

\[ g_R(t) = 1 - \sum_{i=1}^{N} \bar{g}_i \left( 1 - \exp\left( \frac{-t}{\tau_i^G} \right) \right) \]

where \( \gamma(t) \) & \( \tau(t) \)- time-varying shear strain & shear stress
\( G_0 \)- instantaneous shear modulus
\( g_R \)- dimensionless shear modulus
\( N, \tau_i^G, \bar{g}_i \)- material constants
Viscoplastic

- Power-Law Creep (CRP)

\[ \dot{\varepsilon}_e^v = \left\{ A \sigma_e^n \left[ (m + 1) \epsilon_e^v \right]^m \right\}^{1/(m+1)} \]

where \( \dot{\varepsilon}_e^v \) & \( \varepsilon_e^v \) - effective viscous strain & strain rate

\( \sigma_e \) - equivalent deviatoric stress

A, m, n - material constants. If m=0 & n=1, a linear dashpot
Viscoplastic

- Two-Layer Viscoplastic (TL-VP)

\[ \varepsilon = \varepsilon^e + \varepsilon^v = \varepsilon^e + (1 - R)\varepsilon^p + R\varepsilon^v \]

Elastic: \[ \varepsilon^e = R\varepsilon^e_v + (1 - R)\varepsilon^e_p \]

Plastic: \[ f = \sqrt{\frac{3}{2}(S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij}) - \sigma_y} \]

\[ \dot{\alpha}_{ij} = \frac{2}{3}C\dot{\varepsilon}^p_{ij} - \kappa\alpha_{ij}\dot{\varepsilon}^e_e \]

Viscous: \[ \dot{\varepsilon}^v_e = \left\{ A\sigma^e \left[ (m + 1)\varepsilon^v_e \right]^m \right\}^{1/(m+1)} \]

where \( R = \frac{K_v}{K_v + K_p} \)
**Viscoelastic-Plastic**

- **Parallel Rheological Framework (PRF)**

**Elastic:** \( W_T = \sum_{i=0}^{N} s_i W(C_i^e) \)

\[ \sum_{i=0}^{N} s_i = 1 \]

**Plastic:** \( f = \sqrt{\frac{3}{2}} (S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij}) - \sigma_y \)

\[ \dot{\alpha}_{ij} = \frac{2}{3} C \dot{\varepsilon}^p_{ij} - \kappa \alpha_{ij} \dot{\varepsilon}^p_e \]

**Viscous:** \( D^{cr} = \frac{3}{2q} \dot{\varepsilon}^v \bar{\tau} \)

\[ \dot{\varepsilon}^v_e = \{ A \sigma^n_e [(m + 1)\varepsilon^v_e]^m \}^{1/(m+1)} \]
Viscoelastic-Viscoplastic

- Nonlinear Viscoelastic-Viscoplastic (VE-VP)
  - Total Strain
    \[ \varepsilon_{ij} = \varepsilon_{ij}^{ve} + \varepsilon_{ij}^{vp} \]
  - Viscoelastic Model (Schapery)
    \[ \varepsilon^{ve}(t) = \frac{1}{1-D^{ve}} g_0 D_0 \sigma^t + g_1 \int_0^t \Delta D(\psi^t-\psi^\tau) \frac{d(g_2 \sigma^\tau)}{d\tau} d\tau \]
    \[ \Delta D\psi^t = \sum_{n=1}^{5} D_n (1 - \exp(-\lambda_n \psi^t)) \]

Effect of hydrostatic stress

\[ g_0 = \beta_1^j (\sigma_e)^{\beta_2^j} + \beta_3^j, \quad j=+(tension) \text{ or } -(compression) \]

Reversed cyclic response

\[ D^{ve} = \sum_{i=0}^{N} \zeta_i t^i \]
Viscoelastic-Viscoplastic

- Nonlinear Viscoelastic-Viscoplastic (VE-VP)
  - Viscoplastic Model (Perzyna)
    \[
    \dot{\varepsilon}^{vp} = \dot{\lambda}m = \eta \langle \phi(f) \rangle \frac{\partial f}{\partial \sigma_{ij}} = \eta \left( \frac{f}{\sigma_y^0} \right)^N \frac{\partial f}{\partial \sigma_{ij}}
    \]
  - Yield function
    Drucker-Prager Yield + Nonlinear Kinematic Hardening
    \[
    f = A I_1 + \sqrt{\frac{1}{2}} (S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij}) - (1 - D^{vp}B)
    \]
    Reversed cyclic response
    \[
    D^{vp} = 1/t_D
    \]
Numerical Implementation

- Nonlinear Viscoelastic-Viscoplastic (VE-VP)
  - First assume viscoelasticity only.
  - Viscoplasticity is activated when load is beyond yield surface.
  - Damage factors are activated under reversed cyclic loading.
Finite Element Model

- Mesh & Boundary Conditions (Plane Strain)
# Model Calibration

<table>
<thead>
<tr>
<th>Model</th>
<th>Component</th>
<th>Parameters</th>
<th>Test</th>
<th>Stress level</th>
<th>Loading/Recover time</th>
</tr>
</thead>
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<tr>
<td>L-VE</td>
<td>Prony Series</td>
<td>$\tilde{g}_i$, $\tau_i^G$</td>
<td>Creep</td>
<td>20% USS</td>
<td>10,000 s/ 80,000 s</td>
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<tr>
<td>CRP</td>
<td>Viscous</td>
<td>A, m, n</td>
<td>Creep</td>
<td>20% &amp; 50% USS</td>
<td>10,000 s/80,000 s</td>
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<tr>
<td>TL-VP</td>
<td>Plastic</td>
<td>$\sigma_y$, C, $\kappa$</td>
<td>monotonic</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Elastic &amp; Viscous</td>
<td>R, A, m, n</td>
<td>Creep</td>
<td>20% &amp; 50% USS</td>
<td>10,000 s/80,000 s</td>
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<tr>
<td>PRF</td>
<td>Hyperelastic</td>
<td>$C_{10}$, $D_1$</td>
<td>monotonic</td>
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<tr>
<td></td>
<td>Plastic</td>
<td>$\sigma_y$, C, $\kappa$</td>
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<tr>
<td></td>
<td>Viscous</td>
<td>$\gamma_k$, $A_k$, $m_k$, $n_k$</td>
<td>Creep</td>
<td>20% &amp; 50% USS</td>
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<td></td>
<td>Prony series</td>
<td>$D_i$, $\lambda_i$, $J_0$, $B_0$</td>
<td>Creep</td>
<td>20% USS</td>
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<td>Nonlinear VE</td>
<td>$g_1$, $g_2$, a</td>
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<td>$g_0$</td>
<td>monotonic</td>
<td></td>
<td>Varying rates</td>
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<tr>
<td></td>
<td>Viscoplastic</td>
<td>$\eta$, N</td>
<td>Creep</td>
<td>50% USS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Damage</td>
<td>$D^{ve}$, $t_D$</td>
<td>Tensile cyclic</td>
<td>50% USS</td>
<td>Varying load durations</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Reversed cyclic</td>
<td>50% USS</td>
<td>0.5 Hz</td>
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Nonlinear Viscoelastic-Viscoplastic (VE-VP)

\[ \varepsilon_{ve}(t) = \frac{1}{1 - D_{ve}} g_0 D_0 \sigma^t + g_1 \int_0^t \Delta D(\psi^t - \psi^r) \frac{d(g_2 \sigma^r)}{d\tau} d\tau \]

- Effect of hydrostatic stress

\[ g_0 = \beta_1^j (\sigma_e) \beta_2^j + \beta_3^j \]

where \( j = + \) (tension) or \( - \) (compression)

- The effect of hydrostatic stress on VE response can be different in tension and compression.
- The effect can be reflected in the elastic response \( (g_0) \).
VE-VP Modulus Degradation

- Nonlinear Viscoelastic-Viscoplastic (VE-VP)

\[ \varepsilon^{ve}(t) = \frac{1}{1 - D^{ve}} g_0 D_0 \sigma^t + g_1 \int_0^t \Delta D(\psi^t - \psi^\tau) \frac{d(g_2 \sigma^\tau)}{d\tau} d\tau \]

- Reversed cyclic response

\[ D^{ve} = \sum_{i=0}^{N} \zeta_i t^i \]
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Scarf Joint- Creep at 20% & 50% USS
Scarf Joint - Ratcheting with R=0.1, 50% USS

- 0.5 Hz

- 3 Hz
Scarf Joint- Ratcheting with R=-1, 50% USS

• 0.5 Hz
Scarf Joint- Ratcheting with R=-1, 50% USS

- PRF Model
- VE-VP Model
Scarf Joint - Ratcheting with $R=-1$, 50% USS

$3 \text{ Hz}$

$$f = AI_1 + \sqrt{\frac{1}{2} (S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij}) - (1 - D_{vp})B}$$

![Graph showing stress and cycles](image)

![Graph showing shear strain and recovery strain](image)
Scarf Joint- Ratcheting with R=-1, 50% USS

- 5 Hz

- Extensive permanent strain comparable to 0.5 Hz
- Temperature effect?
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Summary & Future Work

• Parameter calibration is highly dependent on model complexity.
• Creep-recovery behavior is well described by the models involving both nonlinear viscoelasticity and plasticity/viscoplasticity.
• Nonlinear viscoelastic-plastic/viscoplastic models (PRF & VE-VP) could describe response to tensile cyclic load.
  • Viscoplasticity is recommended for plastic prediction under varying frequencies and load durations.
• VE-VP model showed good agreement with reversed cyclic test.
  • Pressure-dependent yield criterion results in accumulative plastic deformation.
  • Hydrostatic stress affects viscoelastic response in tension and compression.
  • Damage factors are required for extensive permanent deformation.
• Future Work
  • Investigate reversed cyclic response at higher frequencies and longer load durations (ratcheting and permanent deformation).
  • Modify and validate constitutive model.
THANK YOU

Questions?

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