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# TenCate Advance Composites AS4C 3k-PW Fabric with BT250E-6 Resin Material Allowables Statistical Analysis Report

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## 1. Introduction

This report contains statistical analysis of the TenCate BT250E-6 AS4C 3k-PW Fabric Gr 195gsm 40% RC qualification material property data published in NCAMP Test Report CAM-RP-2015-039 Rev N/C. The lamina material property data have been generated with FAA oversight through FAA Special Project Number TD03019RC-R and also meet the requirements of NCAMP Standard Operating Procedure NSP 100. The test panels, test specimens, and test setups have been conformed by the FAA and the testing has been witnessed by the FAA.

B-Basis values, A-estimates, and B-estimates were calculated using a variety of techniques that are detailed in section two. The qualification material was procured to Erickson Air-Crane (EAC) Material Specification ES0095 Revision B dated May 22, 2013. An equivalent NCAMP Material Specification NMS 250/2 Rev Initial Release dated January 2, 2018 has been created. The qualification test panels were cured in accordance with Erickson Air Crane (EAC) Process Specification ES0098 Rev A dated June 15, 2011. An equivalent NCAMP Process Specification NPS 81250 with baseline "C" Cure Cycle Rev Initial Release dated October 20, 2017 has been created. The panels were fabricated at Advanced Technologies Inc., 875 Middle Ground Blvd. Newport News, VA 23606. The Erickson Air-Crane (EAC) test plan EAC2028 Rev C was used for this qualification program. The testing was performed at the National Institute for Aviation Research (NIAR) in Wichita, Kansas.

Basis numbers are labeled as 'values' when the data meets all the requirements of CMH-17-1G. When those requirements are not met, they will be labeled as 'estimates.' When the data does not meet all requirements, the failure to meet these requirements is reported and the specific requirement(s) the data fails to meet is identified. The method used to compute the basis value is noted for each basis value provided. When appropriate, in addition to the traditional computational methods, values computed using the modified coefficient of variation method is also provided.

The material property data acquisition process is designed to generate basic material property data with sufficient pedigree for submission to Complete Documentation sections of the Composite Materials Handbook (CMH-17-1G).

The NCAMP shared material property database contains material property data of common usefulness to a wide range of aerospace projects. However, the data may not fulfill all the needs of a project. Specific properties, environments, laminate architecture, and loading situations that individual projects need may require additional testing.

The use of NCAMP material and process specifications do not guarantee material or structural performance. Material users should be actively involved in evaluating material performance and quality including, but not limited to, performing regular purchaser quality control tests, performing periodic equivalency/additional testing, participating in material change management activities, conducting statistical process control, and conducting regular supplier audits.

The applicability and accuracy of NCAMP material property data, material allowables, and specifications must be evaluated on case-by-case basis by aircraft companies and certifying

agencies. NCAMP assumes no liability whatsoever, expressed or implied, related to the use of the material property data, material allowables, and specifications.

Part fabricators that wish to utilize the material property data, allowables, and specifications may be able to do so by demonstrating the capability to reproduce the original material properties; a process known as equivalency. More information about this equivalency process including the test statistics and its limitations can be found in Section 6 of DOT/FAA/AR-03/19 and Section 8.4.1 of CMH-17-1G. The applicability of equivalency process must be evaluated on program-by-program basis by the applicant and certifying agency. The applicant and certifying agency must agree that the equivalency test plan along with the equivalency process described in Section 6 of DOT/FAA/AR-03/19 and Section 8.4.1 of CMH-17-1G are adequate for the given program.

Aircraft companies should not use the data published in this report without specifying NCAMP Material Specification NMS 250/2. NMS 250/2 has additional requirements that are listed in its prepreg process control document (PCD), fiber specification, fiber PCD, and other raw material specifications and PCDs which impose essential quality controls on the raw materials and raw material manufacturing equipment and processes. *Aircraft companies and certifying agencies should assume that the material property data published in this report is not applicable when the material is not procured to NCAMP Material Specification NMS 250/2. NMS 250/2 is a free, publicly available, non-proprietary aerospace industry material specification.*

This report is intended for general distribution to the public, either freely or at a price that does not exceed the cost of reproduction (e.g. printing) and distribution (e.g. postage).

### 1.1 Symbols and Abbreviations

Test Property	Abbreviation
Warp Compression	WC
Warp Tension	WT
Fill Compression	FC
Fill Tension	FT
In-Plane Shear	IPS
Short Beam Strength	SBS

**Table 1-1: Test Property Abbreviations**

Test Property	Symbol
Warp Compression Strength	$F_1^{cu}$
Warp Compression Modulus	$E_1^c$
Warp Tension Strength	$F_1^{tu}$
Warp Tension Modulus	$E_1^t$
Warp Tension Poisson's Ratio	$\nu_{12}^t$
Fill Compression Strength	$F_2^{cu}$
Fill Compression Modulus	$E_2^c$
Fill Tension Strength	$F_2^{tu}$
Fill Tension Modulus	$E_2^t$
In-Plane Shear Strength at 5% strain	$F_{12}^{s5\%}$
In-Plane Shear Strength at 0.2% offset	$F_{12}^{s0.2\%}$
In-Plane Shear Modulus	$G_{12}^s$

**Table 1-2: Test Property Symbols**

Environmental Condition	Abbreviation	Temperature
Cold Temperature Dry	CTD	-65°F
Room Temperature Dry	RTD	70°F
Elevated Temperature Dry	ETD	180°F
Elevated Temperature Wet	ETW	180°F

**Table 1-3: Environmental Conditions Abbreviations**

Detailed information about the test methods and conditions used is given in test plan EAC2028 Rev C and NCAMP Test Report CAM-RP-2015-039 Rev N/C.

## 1.2 Pooling Across Environments

When pooling across environments was allowable, the pooled co-efficient of variation was used. CMH17 STATS v2011 r1.1 was used to determine if pooling was allowable and to compute the pooled coefficient of variation for those tests. In these cases, the modified coefficient of variation based on the pooled data was used to compute the basis values.

When pooling across environments was not advisable because the data was not eligible for pooling and engineering judgment indicated there was no justification for overriding the result, then B-Basis values were computed for each environmental condition separately, which are also provided by CMH17 STATS.

## 1.3 Basis Value Computational Process

The general form to compute engineering basis values is:  $\text{basis value} = \bar{X} - kS$  where  $k$  is a factor based on the sample size and the distribution of the sample data. There are many different methods to determine the value of  $k$  in this equation, depending on the sample size and the distribution of the data. In addition, the computational formula used for the standard deviation,  $S$ , may vary depending on the distribution of the data. The details of those different computations and when each should be used are in section 2.0.

## 1.4 Modified Coefficient of Variation (CV) Method

A common problem with new material qualifications is that the initial specimens produced and tested do not contain all of the variability that will be encountered when the material is being produced in larger amounts over a lengthy period of time. This can result in setting basis values that are unrealistically high. The variability as measured in the qualification program is often lower than the actual material variability because of several reasons. The materials used in the qualification programs are usually manufactured within a short period of time, typically 2-3 weeks only, which is not representative of the production material. Some raw ingredients that are used to manufacture the multi-batch qualification materials may actually be from the same production batches or manufactured within a short period of time so the qualification materials, although regarded as multiple batches, may not truly be multiple batches so they are not representative of the actual production material variability.

The modified Coefficient of Variation (CV) used in this report is in accordance with section 8.4.4 of CMH-17-1G. It is a method of adjusting the original basis values downward in anticipation of the expected additional variation. Composite materials are expected to have a CV of at least 6%. The modified coefficient of variation (CV) method increases the measured coefficient of variation when it is below 8% prior to computing basis values. A higher CV will result in lower or more conservative basis values and lower specification limits. The use of the modified CV method is intended for a temporary period of time when there is minimal data available. When a sufficient number of production batches (approximately 8 to 15) have been produced and tested, the as-measured CV may be used so that the basis values and specification limits may be adjusted higher.



The material allowables in this report are calculated using both the as-measured CV and modified CV, so users have the choice of using either one. When the measured CV is greater than 8%, the modified CV method does not change the basis value. NCAMP recommended values make use of the modified CV method when it is appropriate for the data.

When the data fails the Anderson-Darling K-sample test for batch to batch variability or when the data fails the normality test, the modified CV method is not appropriate and no modified CV basis value will be provided. When the ANOVA method is used, it may produce excessively conservative basis values. When appropriate, a single batch or two batch estimate may be provided in addition to the ANOVA estimate.

In some cases a transformation of the data to fit the assumption of the modified CV resulted in the transformed data passing the ADK test and thus the data can be pooled only for the modified CV method.

NCAMP recommends that if a user decides to use the basis values that are calculated from as-measured CV, the specification limits and control limits be calculated with as-measured CV also. Similarly, if a user decides to use the basis values that are calculated from modified CV, the specification limits and control limits be calculated with modified CV also. This will ensure that the link between material allowables, specification limits, and control limits is maintained.

## 2. Background

Statistical computations are performed with CMH17 STATS. Pooling across environments will be used whenever it is permissible according to CMH-17-1G guidelines. If pooling is not permissible, the results of a single point analysis provided by CMH17 STATS is included instead. If the data does not meet CMH-17-1G requirements for a single point analysis, estimates are created by a variety of methods depending on which is most appropriate for the dataset available. Specific procedures used are presented in the individual sections where the data is presented.

### 2.1 CMH17 STATS Statistical Formulas and Computations

This section contains the details of the specific formulas CMH17 STATS program uses in its computations.

#### 2.1.1 Basic Descriptive Statistics

The basic descriptive statistics shown are computed according to the usual formulas, which are shown below:

<b>Mean:</b>	$\bar{X} = \sum_{i=1}^n \frac{X_i}{n}$	<b>Equation 1</b>
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<b>Std. Dev.:</b>	$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2}$	<b>Equation 2</b>
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<b>% Co. Variation:</b>	$\frac{S}{\bar{X}} \times 100$	<b>Equation 3</b>
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Where  $n$  refers to the number of specimens in the sample and  $X_i$  refers to the individual specimen measurements.

#### 2.1.2 Statistics for Pooled Data

Prior to computing statistics for the pooled dataset, the data is normalized to a mean of one by dividing each value by the mean of all the data for that condition. This transformation does not affect the coefficients of variation for the individual conditions.

##### 2.1.2.1 Pooled Standard Deviation

The formula to compute a pooled standard deviation is given below:

<b>Pooled Std. Dev.:</b>	$S_p = \sqrt{\frac{\sum_{i=1}^k (n_i - 1) S_i^2}{\sum_{i=1}^k (n_i - 1)}}$	<b>Equation 4</b>
--------------------------	--	-------------------

Where  $k$  refers to the number of batches,  $S_i$  indicates the standard deviation of  $i^{\text{th}}$  sample, and  $n_i$  refers to the number of specimens in the  $i^{\text{th}}$  sample.

**2.1.2.2 Pooled Coefficient of Variation**

Since the mean for the normalized data is 1.0 for each condition, the pooled normalized data also has a mean of one. The coefficient of variation for the pooled normalized data is the pooled standard deviation divided by the pooled mean, as in equation 3. Since the mean for the pooled normalized data is one, the pooled coefficient of variation is equal to the pooled standard deviation of the normalized data.

$$\text{Pooled Coefficient of Variation} = \frac{S_p}{1} = S_p \tag{Equation 5}$$

**2.1.3 Basis Value Computations**

Basis values are computed using the mean and standard deviation for that environment, as follows: The mean is always the mean for the environment, but if the data meets all requirements for pooling,  $S_p$  can be used in place of the standard deviation for the environment,  $S$ .

$$\begin{aligned} \text{Basis Values:} \quad A\text{-basis} &= \bar{X} - K_a S \\ B\text{-basis} &= \bar{X} - K_b S \end{aligned} \tag{Equation 6}$$

**2.1.3.1 K-factor computations**

$K_a$  and  $K_b$  are computed according to the methodology documented in section 8.3.5 of CMH-17-1G. The approximation formulas are given below:

$$K_a = \frac{2.3263}{\sqrt{q(f)}} + \sqrt{\frac{1}{c_A(f) \cdot n_j} + \left(\frac{b_A(f)}{2c_A(f)}\right)^2} - \frac{b_A(f)}{2c_A(f)} \tag{Equation 7}$$

$$K_b = \frac{1.2816}{\sqrt{q(f)}} + \sqrt{\frac{1}{c_B(f) \cdot n_j} + \left(\frac{b_B(f)}{2c_B(f)}\right)^2} - \frac{b_B(f)}{2c_B(f)} \tag{Equation 8}$$

Where

$r$  = the number of environments being pooled together

$n_j$  = number of data values for environment  $j$

$$N = \sum_{j=1}^r n_j$$

$$f = N - r$$

$$q(f) = 1 - \frac{2.323}{\sqrt{f}} + \frac{1.064}{f} + \frac{0.9157}{f\sqrt{f}} - \frac{0.6530}{f^2} \tag{Equation 9}$$

$$b_B(f) = \frac{1.1372}{\sqrt{f}} - \frac{0.49162}{f} + \frac{0.18612}{f\sqrt{f}} \quad \text{Equation 10}$$

$$c_B(f) = 0.36961 + \frac{0.0040342}{\sqrt{f}} - \frac{0.71750}{f} + \frac{0.19693}{f\sqrt{f}} \quad \text{Equation 11}$$

$$b_A(f) = \frac{2.0643}{\sqrt{f}} - \frac{0.95145}{f} + \frac{0.51251}{f\sqrt{f}} \quad \text{Equation 12}$$

$$c_A(f) = 0.36961 + \frac{0.0026958}{\sqrt{f}} - \frac{0.65201}{f} + \frac{0.011320}{f\sqrt{f}} \quad \text{Equation 13}$$

**2.1.4 Modified Coefficient of Variation**

The coefficient of variation is modified according to the following rules:

$$\text{Modified CV} = CV^* = \begin{cases} .06 & \text{if } CV < .04 \\ \frac{CV}{2} + .04 & \text{if } .04 \leq CV < .08 \\ CV & \text{if } CV \geq .08 \end{cases} \quad \text{Equation 14}$$

This is converted to percent by multiplying by 100%.

CV\* is used to compute a modified standard deviation S\*.

$$S^* = CV^* \cdot \bar{X} \quad \text{Equation 15}$$

To compute the pooled standard deviation based on the modified CV:

$$S_p^* = \sqrt{\frac{\sum_{i=1}^k ((n_i - 1)(CV_i^* \cdot \bar{X}_i)^2)}{\sum_{i=1}^k (n_i - 1)}} \quad \text{Equation 16}$$

The A-basis and B-basis values under the assumption of the modified CV method are computed by replacing S with S\*

**2.1.4.1 Transformation of data based on Modified CV**

In order to determine if the data would pass the diagnostic tests under the assumption of the modified CV, the data must be transformed such that the batch means remain the same while the standard deviation of transformed data (all batches) matches the modified standard deviation.

To accomplish this requires a transformation in two steps:

Step 1: Apply the modified CV rules to each batch and compute the modified standard deviation  $S_i^* = CV^* \cdot \bar{X}_i$  for each batch. Transform the individual data values ( $X_{ij}$ ) in each batch as follows:

$$X'_{ij} = C_i (X_{ij} - \bar{X}_i) + \bar{X}_i \tag{Equation 17}$$

$$C_i = \frac{S_i^*}{S_i} \tag{Equation 18}$$

Run the Anderson-Darling k-sample test for batch equivalence (see section 2.1.6) on the transformed data. If it passes, proceed to step 2. If not, stop. The data cannot be pooled.

Step 2: Another transformation is needed as applying the modified CV to each batch leads to a larger CV for the combined data than when applying the modified CV rules to the combined data (due to the addition of between batch variation when combining data from multiple batches). In order to alter the data to match  $S^*$ , the transformed data is transformed again, this time setting using the same value of  $C'$  for all batches.

$$X''_{ij} = C' (X'_{ij} - \bar{X}_i) + \bar{X}_i \tag{Equation 19}$$

$$C' = \sqrt{\frac{SSE^*}{SSE'}} \tag{Equation 20}$$

$$SSE^* = (n-1)(CV^* \cdot \bar{X})^2 - \sum_{i=1}^k n_i (\bar{X}_i - \bar{X})^2 \tag{Equation 21}$$

$$SSE' = \sum_{i=1}^k \sum_{j=1}^{n_i} (X'_{ij} - \bar{X}_i)^2 \tag{Equation 22}$$

Once this second transformation has been completed, the k-sample Anderson Darling test for batch equivalence can be run on the transformed data to determine if the modified co-efficient of variation will permit pooling of the data.

### 2.1.5 Determination of Outliers

All outliers are identified in text and graphics. If an outlier is removed from the dataset, it will be specified and the reason why will be documented in the text. Outliers are identified using the Maximum Normed Residual Test for Outliers as specified in section 8.3.3 of CMH-17-1G.

$$MNR = \frac{\max_{all\ i} |X_i - \bar{X}|}{S}, i = 1K\ n \tag{Equation 23}$$

$$C = \frac{n-1}{\sqrt{n}} \sqrt{\frac{t^2}{n-2+t^2}} \tag{Equation 24}$$

where t is the  $1 - \frac{0.5}{2n}$  quartile of a t distribution with n-2 degrees of freedom, n being the total number of data values.

If  $MNR > C$ , then the  $X_i$  associated with the MNR is considered to be an outlier. If an outlier exists, then the  $X_i$  associated with the MNR is dropped from the dataset and the MNR procedure is applied again. This process is repeated until no outliers are detected. Additional information on this procedure can be found in references 1 and 2.

**2.1.6 The k-Sample Anderson Darling Test for Batch Equivalency**

The k-sample Anderson-Darling test is a nonparametric statistical procedure that tests the hypothesis that the populations from which two or more groups of data were drawn are identical. The distinct values in the combined data set are ordered from smallest to largest, denoted  $z(1), z(2), \dots, z(L)$ , where  $L$  will be less than  $n$  if there are tied observations. These rankings are used to compute the test statistic.

The k-sample Anderson-Darling test statistic is:

$$ADK = \frac{n-1}{n^2(k-1)} \sum_{i=1}^k \left[ \frac{1}{n_i} \sum_{j=1}^L h_j \frac{(nF_{ij} - n_i H_j)^2}{H_j(n - H_j) - \frac{nh_j}{4}} \right] \tag{Equation 25}$$

Where

$n_i$  = the number of test specimens in each batch

$n = n_1 + n_2 + \dots + n_k$

$h_j$  = the number of values in the combined samples equal to  $z(j)$

$H_j$  = the number of values in the combined samples less than  $z(j)$  plus  $\frac{1}{2}$  the number of values in the combined samples equal to  $z(j)$

$F_{ij}$  = the number of values in the  $i^{th}$  group which are less than  $z(j)$  plus  $\frac{1}{2}$  the number of values in this group which are equal to  $z(j)$ .

The critical value for the test statistic at  $1-\alpha$  level is computed:

$$ADC = 1 + \sigma_n \left[ z_\alpha + \frac{0.678}{\sqrt{k-1}} - \frac{0.362}{k-1} \right] \tag{Equation 26}$$

This formula is based on the formula in reference 3 at the end of section 5, using a Taylor's expansion to estimate the critical value via the normal distribution rather than using the t distribution with  $k-1$  degrees of freedom.

$$\sigma_n^2 = VAR(ADK) = \frac{an^3 + bn^2 + cn + d}{(n-1)(n-2)(n-3)(k-1)^2} \tag{Equation 27}$$

With

$$\begin{aligned}
 a &= (4g - 6)(k - 1) + (10 - 6g)S \\
 b &= (2g - 4)k^2 + 8Tk + (2g - 14T - 4)S - 8T + 4g - 6 \\
 c &= (6T + 2g - 2)k^2 + (4T - 4g + 6)k + (2T - 6)S + 4T \\
 d &= (2T + 6)k^2 - 4Tk \\
 S &= \sum_{i=1}^k \frac{1}{n_i} \\
 T &= \sum_{i=1}^{n-1} \frac{1}{i} \\
 g &= \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \frac{1}{(n-i)j}
 \end{aligned}$$

The data is considered to have failed this test (i.e. the batches are not from the same population) when the test statistic is greater than the critical value. For more information on this procedure, see reference 3.

### 2.1.7 The Anderson Darling Test for Normality

**Normal Distribution:** A two parameter ( $\mu, \sigma$ ) family of probability distributions for which the probability that an observation will fall between  $a$  and  $b$  is given by the area under the curve between  $a$  and  $b$ :

$$F(x) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad \text{Equation 28}$$

A normal distribution with parameters ( $\mu, \sigma$ ) has population mean  $\mu$  and variance  $\sigma^2$ .

The normal distribution is considered by comparing the cumulative normal distribution function that best fits the data with the cumulative distribution function of the data. Let

$$z_{(i)} = \frac{x_{(i)} - \bar{x}}{s}, \quad \text{for } i = 1, K, n \quad \text{Equation 29}$$

where  $x_{(i)}$  is the smallest sample observation,  $\bar{x}$  is the sample average, and  $s$  is the sample standard deviation.

The Anderson Darling test statistic (AD) is:

$$AD = \sum_{i=1}^n \frac{1-2i}{n} \left\{ \ln [F_0(z_{(i)})] + \ln [1 - F_0(z_{(n+1-i)})] \right\} - n \quad \text{Equation 30}$$

Where  $F_0$  is the standard normal distribution function. The observed significance level (OSL) is

$$OSL = \frac{1}{1 + e^{-0.48 + 0.78 \ln(AD^*) + 4.58 AD^*}}, \quad AD^* = \left( 1 + \frac{0.2}{\sqrt{n}} \right) AD \quad \text{Equation 31}$$

This OSL measures the probability of observing an Anderson-Darling statistic at least as extreme as the value calculated if, in fact, the data are a sample from a normal population. If  $OSL > 0.05$ , the data is considered sufficiently close to a normal distribution.

**2.1.8 Levene’s Test for Equality of Coefficient of Variation**

Levene’s test performs an Analysis of Variance on the absolute deviations from their sample medians. The absolute value of the deviation from the median is computed for each data value.

$w_{ij} = |y_{ij} - \hat{\mu}_i|$  An F-test is then performed on the transformed data values as follows:

$$F = \frac{\sum_{i=1}^k n_i (\bar{w}_i - \bar{w})^2 / (k - 1)}{\sum_{i=1}^k \sum_{j=1}^{n_i} (w_{ij} - \bar{w}_i)^2 / (n - k)} \tag{Equation 32}$$

If this computed F statistic is less than the critical value for the F-distribution having k-1 numerator and n-k denominator degrees of freedom at the 1- $\alpha$  level of confidence, then the data is not rejected as being too different in terms of the co-efficient of variation. ASAP provides the appropriate critical values for F at  $\alpha$  levels of 0.10, 0.05, 0.025, and 0.01. For more information on this procedure, see references 4 and 5.

**2.2 STAT-17 and CMH17 STATS Statistical Formulas and Computations**

This section contains the details of the specific formulas STAT-17 and CMH17 STATS uses in their computations.

The basic descriptive statistics, the maximum normed residual (MNR) test for outliers, and the Anderson Darling K-sample test for batch variability are the same as with ASAP – see sections 2.1.1, 2.1.3.1, and 2.1.5.

Outliers must be dispositioned before checking any other test results. The results of the Anderson Darling k-Sample (ADK) Test for batch equivalency must be checked. If the data passes the ADK test, then the appropriate distribution is determined. If it does not pass the ADK test, then the ANOVA procedure is the only approach remaining that will result in basis values that meet the requirements of CMH-17-1G.

**2.2.1 Distribution Tests**

In addition to testing for normality using the Anderson-Darling test (see 2.1.7); Stat17 also tests to see if the Weibull or Lognormal distribution is a good fit for the data.

Each distribution is considered using the Anderson-Darling test statistic which is sensitive to discrepancies in the tail regions. The Anderson-Darling test compares the cumulative distribution function for the distribution of interest with the cumulative distribution function of the data.



An observed significance level (OSL) based on the Anderson-Darling test statistic is computed for each test. The OSL measures the probability of observing an Anderson-Darling test statistic at least as extreme as the value calculated if the distribution under consideration is in fact the underlying distribution of the data. In other words, the OSL is the probability of obtaining a value of the test statistic at least as large as that obtained if the hypothesis that the data are actually from the distribution being tested is true. If the OSL is less than or equal to 0.05, then the assumption that the data are from the distribution being tested is rejected with at most a five percent risk of being in error.

If the normal distribution has an OSL greater than 0.05, then the data is assumed to be from a population with a normal distribution. If not, then if either the Weibull or lognormal distributions has an OSL greater than 0.05, then one of those can be used. If neither of these distributions has an OSL greater than 0.05, a non-parametric approach is used.

In what follows, unless otherwise noted, the sample size is denoted by  $n$ , the sample observations by  $x_1, \dots, x_n$ , and the sample observations ordered from least to greatest by  $x_{(1)}, \dots, x_{(n)}$ .

**2.2.2 Computing Normal Distribution Basis Values**

Stat17 uses a table of values for the k-factors (shown in Table 2-1) when the sample size is less than 16 and a slightly different formula than ASAP to compute approximate k-values for the normal distribution when the sample size is 16 or larger.

Norm. Dist. k Factors for N<16		
N	B-basis	A-basis
2	20.581	37.094
3	6.157	10.553
4	4.163	7.042
5	3.408	5.741
6	3.007	5.062
7	2.756	4.642
8	2.583	4.354
9	2.454	4.143
10	2.355	3.981
11	2.276	3.852
12	2.211	3.747
13	2.156	3.659
14	2.109	3.585
15	2.069	3.520

**Table 2-1: K factors for normal distribution**

**2.2.2.1 One-sided B-basis tolerance factors,  $k_B$ , for the normal distribution when sample size is greater than 15.**

The exact computation of  $k_B$  values is  $1/\sqrt{n}$  times the 0.95th quantile of the noncentral t-distribution with noncentrality parameter  $1.282\sqrt{n}$  and  $n - 1$  degrees of freedom. Since this in not a calculation that Excel can handle, the following approximation to the  $k_B$  values is used:

$$k_B \approx 1.282 + \exp\{0.958 - 0.520 \ln(n) + 3.19/n\} \tag{Equation 33}$$

This approximation is accurate to within 0.2% of the tabulated values for sample sizes greater than or equal to 16.

**2.2.2.2 One-sided A-basis tolerance factors,  $k_A$ , for the normal distribution**

The exact computation of  $k_B$  values is  $1/\sqrt{n}$  times the 0.95th quantile of the noncentral t-distribution with noncentrality parameter  $2.326\sqrt{n}$  and  $n - 1$  degrees of freedom (Reference 11). Since this is not a calculation that Excel can handle easily, the following approximation to the  $k_B$  values is used:

$$k_A \approx 2.326 + \exp\{1.34 - 0.522 \ln(n) + 3.87/n\} \tag{Equation 34}$$

This approximation is accurate to within 0.2% of the tabulated values for sample sizes greater than or equal to 16.

**2.2.2.3 Two-parameter Weibull Distribution**

A probability distribution for which the probability that a randomly selected observation from this population lies between  $a$  and  $b$  ( $0 < a < b < \infty$ ) is given by

$$e^{-(a/\alpha)^\beta} - e^{-(b/\alpha)^\beta} \tag{Equation 35}$$

where  $\alpha$  is called the scale parameter and  $\beta$  is called the shape parameter.

In order to compute a check of the fit of a data set to the Weibull distribution and compute basis values assuming Weibull, it is first necessary to obtain estimates of the population shape and scale parameters (Section 2.2.2.3.1). Calculations specific to the goodness-of-fit test for the Weibull distribution are provided in section 2.2.2.3.2.

**2.2.2.3.1 Estimating Weibull Parameters**

This section describes the *maximum likelihood* method for estimating the parameters of the two-parameter Weibull distribution. The maximum-likelihood estimates of the shape and scale parameters are denoted  $\hat{\beta}$  and  $\hat{\alpha}$ . The estimates are the solution to the pair of equations:

$$\hat{\alpha}\hat{\beta}n - \frac{\hat{\beta}}{\hat{\alpha}^{\hat{\beta}-1}} \sum_{i=1}^n x_i^{\hat{\beta}} = 0 \tag{Equation 36}$$

$$\frac{n}{\hat{\beta}} - n \ln \hat{\alpha} + \sum_{i=1}^n \ln x_i - \sum_{i=1}^n \left[ \frac{x_i}{\hat{\alpha}} \right]^{\hat{\beta}} (\ln x_i - \ln \hat{\alpha}) = 0 \tag{Equation 37}$$

Stat17 solves these equations numerically for  $\hat{\beta}$  and  $\hat{\alpha}$  in order to compute basis values.

**2.2.2.3.2 Goodness-of-fit test for the Weibull distribution**

The two-parameter Weibull distribution is considered by comparing the cumulative Weibull distribution function that best fits the data with the cumulative distribution function of the data. Using the shape and scale parameter estimates from section 2.2.2.3.1, let

$$z_{(i)} = \left[ x_{(i)} / \hat{\alpha} \right]^{\hat{\beta}}, \text{ for } i = 1, K, n \tag{Equation 38}$$

The Anderson-Darling test statistic is

$$AD = \sum_{i=1}^n \frac{1-2i}{n} \left[ \ln \left[ 1 - \exp(-z_{(i)}) \right] - z_{(n+1-i)} \right] - n \tag{Equation 39}$$

and the observed significance level is

$$OSL = 1 / \left\{ 1 + \exp[-0.10 + 1.24 \ln(AD^*) + 4.48 AD^*] \right\} \tag{Equation 40}$$

where

$$AD^* = \left( 1 + \frac{0.2}{\sqrt{n}} \right) AD \tag{Equation 41}$$

This OSL measures the probability of observing an Anderson-Darling statistic at least as extreme as the value calculated if in fact the data is a sample from a two-parameter Weibull distribution. If  $OSL \leq 0.05$ , one may conclude (at a five percent risk of being in error) that the population does not have a two-parameter Weibull distribution. Otherwise, the hypothesis that the population has a two-parameter Weibull distribution is not rejected. For further information on these procedures, see reference 6.

**2.2.2.3.3 Basis value calculations for the Weibull distribution**

For the two-parameter Weibull distribution, the B-basis value is

$$B = \hat{q} e^{\left( \frac{-V}{\hat{\beta} \sqrt{n}} \right)} \tag{Equation 42}$$

where

$$\hat{q} = \hat{\alpha} (0.10536)^{1/\hat{\beta}} \tag{Equation 43}$$

To calculate the A-basis value, substitute the equation below for the equation above.

$$\hat{q} = \hat{\alpha} (0.01005)^{1/\hat{\beta}} \tag{Equation 44}$$

V is the value in Table 2-2. when the sample size is less than 16. For sample sizes of 16 or larger, a numerical approximation to the V values is given in the two equations immediately below.

$$V_B \approx 3.803 + \exp \left[ 1.79 - 0.516 \ln(n) + \frac{5.1}{n-1} \right] \tag{Equation 45}$$

$$V_A \approx 6.649 + \exp \left[ 2.55 - 0.526 \ln(n) + \frac{4.76}{n} \right] \quad \text{Equation 46}$$

This approximation is accurate within 0.5% of the tabulated values for n greater than or equal to 16.

Weibull Dist. K Factors for N<16		
N	B-basis	A-basis
2	690.804	1284.895
3	47.318	88.011
4	19.836	36.895
5	13.145	24.45
6	10.392	19.329
7	8.937	16.623
8	8.047	14.967
9	7.449	13.855
10	6.711	12.573
11	6.477	12.093
12	6.286	11.701
13	6.127	11.375
14	5.992	11.098
15	5.875	10.861

Table 2-2: Weibull Distribution Basis Value Factors

**2.2.2.4 Lognormal Distribution**

A probability distribution for which the probability that an observation selected at random from this population falls between a and b ( $0 < a < b < \infty$ ) is given by the area under the normal distribution between  $\ln(a)$  and  $\ln(b)$ .

The lognormal distribution is a positively skewed distribution that is simply related to the normal distribution. If something is lognormally distributed, then its logarithm is normally distributed. The natural (base e) logarithm is used.

**2.2.2.4.1 Goodness-of-fit test for the Lognormal distribution**

In order to test the goodness-of-fit of the lognormal distribution, take the logarithm of the data and perform the Anderson-Darling test for normality from Section 2.1.7. Using the natural logarithm, replace the linked equation above with linked equation below:

$$z_{(i)} = \frac{\ln(x_{(i)}) - \bar{x}_L}{s_L}, \quad \text{for } i = 1, K, n \quad \text{Equation 47}$$

where  $x_{(i)}$  is the  $i^{\text{th}}$  smallest sample observation,  $\bar{x}_L$  and  $s_L$  are the mean and standard deviation of the  $\ln(x_i)$  values.

The Anderson-Darling statistic is then computed using the linked equation above and the observed significance level (OSL) is computed using the linked equation above. This OSL measures the probability of observing an Anderson-Darling statistic at least as extreme as the

value calculated if in fact the data are a sample from a lognormal distribution. If  $OSL \leq 0.05$ , one may conclude (at a five percent risk of being in error) that the population is not lognormally distributed. Otherwise, the hypothesis that the population is lognormally distributed is not rejected. For further information on these procedures, see reference 6.

**2.2.2.4.2 Basis value calculations for the Lognormal distribution**

If the data set is assumed to be from a population with a lognormal distribution, basis values are calculated using the equation above in section 2.1.3. However, the calculations are performed using the logarithms of the data rather than the original observations. The computed basis values are then transformed back to the original units by applying the inverse of the log transformation.

**2.2.3 Non-parametric Basis Values**

Non-parametric techniques do not assume any particularly underlying distribution for the population the sample comes from. It does require that the batches be similar enough to be grouped together, so the ADK test must have a positive result. While it can be used instead of assuming the normal, lognormal or Weibull distribution, it typically results in lower basis values. One of following two methods should be used, depending on the sample size.

**2.2.3.1 Non-parametric Basis Values for large samples**

The required sample sizes for this ranking method differ for A and B basis values. A sample size of at least 29 is needed for the B-basis value while a sample size of 299 is required for the A-basis.

To calculate a B-basis value for  $n > 28$ , the value of r is determined with the following formulas:

For B-basis values:

$$r_B = \frac{n}{10} - 1.645 \sqrt{\frac{9n}{100}} + 0.23 \tag{Equation 48}$$

For A-Basis values:

$$r_A = \frac{n}{100} - 1.645 \sqrt{\frac{99n}{10,000}} + 0.29 + \frac{19.1}{n} \tag{Equation 49}$$

The formula for the A-basis values should be rounded to the nearest integer. This approximation is exact for most values and for a small percentage of values (less than 0.2%), the approximation errs by one rank on the conservative side.

The B-basis value is the  $r_B^{\text{th}}$  lowest observation in the data set, while the A-basis values are the  $r_A^{\text{th}}$  lowest observation in the data set. For example, in a sample of size  $n = 30$ , the lowest ( $r = 1$ ) observation is the B-basis value. Further information on this procedure may be found in reference 7.

### 2.2.3.2 Non-parametric Basis Values for small samples

The Hanson-Koopmans method (references 8 and 9) is used for obtaining a B-basis value for sample sizes not exceeding 28 and A-basis values for sample sizes less than 299. This procedure requires the assumption that the observations are a random sample from a population for which the logarithm of the cumulative distribution function is concave, an assumption satisfied by a large class of probability distributions. There is substantial empirical evidence that suggests that composite strength data satisfies this assumption.

The Hanson-Koopmans B-basis value is:

$$B = x_{(r)} \left[ \frac{x_{(1)}}{x_{(r)}} \right]^k \quad \text{Equation 50}$$

The A-basis value is:

$$A = x_{(n)} \left[ \frac{x_{(1)}}{x_{(n)}} \right]^k \quad \text{Equation 51}$$

where  $x_{(n)}$  is the largest data value,  $x_{(1)}$  is the smallest, and  $x_{(r)}$  is the  $r^{\text{th}}$  largest data value. The values of  $r$  and  $k$  depend on  $n$  and are listed in Table 2-3. This method is not used for the B-basis value when  $x_{(r)} = x_{(1)}$ .

The Hanson-Koopmans method can be used to calculate A-basis values for  $n$  less than 299. Find the value  $k_A$  corresponding to the sample size  $n$  in Table 2-4. For an A-basis value that meets all the requirements of CMH-17-1G, there must be at least five batches represented in the data and at least 55 data points. For a B-basis value, there must be at least three batches represented in the data and at least 18 data points.

B-Basis Hanson-Koopmans Table		
n	r	k
2	2	35.177
3	3	7.859
4	4	4.505
5	4	4.101
6	5	3.064
7	5	2.858
8	6	2.382
9	6	2.253
10	6	2.137
11	7	1.897
12	7	1.814
13	7	1.738
14	8	1.599
15	8	1.540
16	8	1.485
17	8	1.434
18	9	1.354
19	9	1.311
20	10	1.253
21	10	1.218
22	10	1.184
23	11	1.143
24	11	1.114
25	11	1.087
26	11	1.060
27	11	1.035
28	12	1.010

Table 2-3: B-Basis Hanson-Koopmans Table

A-Basis Hanson-Koopmans Table					
n	k	n	k	n	k
2	80.00380	38	1.79301	96	1.32324
3	16.91220	39	1.77546	98	1.31553
4	9.49579	40	1.75868	100	1.30806
5	6.89049	41	1.74260	105	1.29036
6	5.57681	42	1.72718	110	1.27392
7	4.78352	43	1.71239	115	1.25859
8	4.25011	44	1.69817	120	1.24425
9	3.86502	45	1.68449	125	1.23080
10	3.57267	46	1.67132	130	1.21814
11	3.34227	47	1.65862	135	1.20620
12	3.15540	48	1.64638	140	1.19491
13	3.00033	49	1.63456	145	1.18421
14	2.86924	50	1.62313	150	1.17406
15	2.75672	52	1.60139	155	1.16440
16	2.65889	54	1.58101	160	1.15519
17	2.57290	56	1.56184	165	1.14640
18	2.49660	58	1.54377	170	1.13801
19	2.42833	60	1.52670	175	1.12997
20	2.36683	62	1.51053	180	1.12226
21	2.31106	64	1.49520	185	1.11486
22	2.26020	66	1.48063	190	1.10776
23	2.21359	68	1.46675	195	1.10092
24	2.17067	70	1.45352	200	1.09434
25	2.13100	72	1.44089	205	1.08799
26	2.09419	74	1.42881	210	1.08187
27	2.05991	76	1.41724	215	1.07595
28	2.02790	78	1.40614	220	1.07024
29	1.99791	80	1.39549	225	1.06471
30	1.96975	82	1.38525	230	1.05935
31	1.94324	84	1.37541	235	1.05417
32	1.91822	86	1.36592	240	1.04914
33	1.89457	88	1.35678	245	1.04426
34	1.87215	90	1.34796	250	1.03952
35	1.85088	92	1.33944	275	1.01773
36	1.83065	94	1.33120	299	1.00000
37	1.81139				

Table 2-4: A-Basis Hanson-Koopmans Table

**2.2.4 Analysis of Variance (ANOVA) Basis Values**

ANOVA is used to compute basis values when the batch to batch variability of the data does not pass the ADK test. Since ANOVA makes the assumption that the different batches have equal variances, the data is checked to make sure the assumption is valid. Levene’s test for equality of variance is used (see section 2.1.8). If the dataset fails Levene’s test, the basis values computed are likely to be conservative. Thus this method can still be used but the values produced will be listed as estimates.



**2.2.4.1 Calculation of basis values using ANOVA**

The following calculations address batch-to-batch variability. In other words, the only grouping is due to batches and the k-sample Anderson-Darling test (Section 2.1.6) indicates that the batch to batch variability is too large to pool the data. The method is based on the one-way analysis of variance random-effects model, and the procedure is documented in reference 10.

ANOVA separates the total variation (called the sum of squares) of the data into two sources: between batch variation and within batch variation.

First, statistics are computed for each batch, which are indicated with a subscript  $(n_i, \bar{x}_i, s_i^2)$  while statistics that were computed with the entire dataset do not have a subscript. Individual data values are represented with a double subscript, the first number indicated the batch and the second distinguishing between the individual data values within the batch.  $k$  stands for the number of batches in the analysis. With these statistics, the Sum of Squares Between batches (SSB) and the Total Sum of Squares (SST) are computed:

$$SSB = \sum_{i=1}^k n_i \bar{x}_i^2 - n \bar{x}^2 \tag{Equation 52}$$

$$SST = \sum_{i=1}^k \sum_{j=1}^{n_i} x_{ij}^2 - n \bar{x}^2 \tag{Equation 53}$$

The within-batch, or error, sum of squares (SSE) is computed by subtraction

$$SSE = SST - SSB \tag{Equation 54}$$

Next, the mean sums of squares are computed:

$$MSB = \frac{SSB}{k - 1} \tag{Equation 55}$$

$$MSE = \frac{SSE}{n - k} \tag{Equation 56}$$

Since the batches need not have equal numbers of specimens, an ‘effective batch size,’ is defined as

$$n' = \frac{n - \frac{1}{n} \sum_{i=1}^k n_i^2}{k - 1} \tag{Equation 57}$$

Using the two mean squares and the effective batch size, an estimate of the population standard deviation is computed:

$$S = \sqrt{\frac{MSB}{n'} + \left(\frac{n' - 1}{n'}\right) MSE} \tag{Equation 58}$$

Two k-factors are computed using the methodology of section 2.2.2 using a sample size of n (denoted k<sub>0</sub>) and a sample size of k (denoted k<sub>1</sub>). Whether this value is an A- or B-basis value depends only on whether k<sub>0</sub> and k<sub>1</sub> are computed for A or B-basis values.

Denote the ratio of mean squares by

$$u = \frac{MSB}{MSE} \tag{Equation 59}$$

If u is less than one, it is set equal to one. The tolerance limit factor is

$$T = \frac{k_0 - \frac{k_1}{\sqrt{n'}} + (k_1 - k_0) \sqrt{\frac{u}{u + n' - 1}}}{1 - \frac{1}{\sqrt{n'}}} \tag{Equation 60}$$

The basis value is  $\bar{x} - TS$ .

The ANOVA method can produce extremely conservative basis values when a small number of batches are available. Therefore, when less than five (5) batches are available and the ANOVA method is used, the basis values produced will be listed as estimates.

### 2.3 Single Batch and Two Batch Estimates using Modified CV

This method has not been approved for use by the CMH-17 organization. Values computed in this manner are estimates only. It is used only when fewer than three batches are available and no valid B-basis value could be computed using any other method. The estimate is made using the mean of the data and setting the coefficient of variation to 8 percent if it was less than that. A modified standard deviation (S<sub>adj</sub>) was computed by multiplying the mean by 0.08 and computing the A and B-basis values using this inflated value for the standard deviation.

$$\text{Estimated B-Basis} = \bar{X} - k_b S_{adj} = \bar{X} - k_b \cdot 0.08 \cdot \bar{X} \tag{Equation 61}$$

### 2.4 Lamina Variability Method (LVM)

This method has not been approved for use by the CMH-17 organization. Values computed in this manner are estimates only. It is used only when the sample size is less than 16 and no valid B-basis value could be computed using any other method. The prime assumption for applying the LVM is that the intrinsic strength variability of the laminate (small) dataset is no greater than the strength variability of the lamina (large) dataset. This assumption was tested and found to be reasonable for composite materials as documented by Tomblin and Seneviratne [12].

To compute the estimate, the coefficients of variation (CVs) of laminate data are paired with lamina CV's for the same loading condition and environmental condition. For example, the 0° compression lamina CV CTD condition is used with open hole compression CTD condition. Bearing and in-plane shear laminate CV's are paired with 0° compression lamina CV's.

However, if the laminate CV is larger than the corresponding lamina CV, the larger laminate CV value is used.

The LVM B-basis value is then computed as:

$$\text{LVM Estimated B-Basis} = \bar{X}_1 - K_{(N_1, N_2)} \cdot \bar{X}_1 \cdot \max(CV_1, CV_2) \quad \text{Equation 62}$$

When used in conjunction with the modified CV approach, a minimum value of 8% is used for the CV.

$$\text{Mod CV LVM Estimated B-Basis} = \bar{X}_1 - K_{(N_1, N_2)} \cdot \bar{X}_1 \cdot \text{Max}(8\%, CV_1, CV_2) \quad \text{Equation 63}$$

With:

- $\bar{X}_1$  the mean of the laminate (small dataset)
- $N_1$  the sample size of the laminate (small dataset)
- $N_2$  the sample size of the lamina (large dataset)
- $CV_1$  is the coefficient of variation of the laminate (small dataset)
- $CV_2$  is the coefficient of variation of the lamina (large dataset)
- $K_{(N_1, N_2)}$  is given in Table 2-5

		N1														
		2	3	4	5	6	7	8	9	10	11	12	13	14	15	
N1+N2-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	3	4.508	0	0	0	0	0	0	0	0	0	0	0	0	0	
	4	3.827	3.607	0	0	0	0	0	0	0	0	0	0	0	0	
	5	3.481	3.263	3.141	0	0	0	0	0	0	0	0	0	0	0	
	6	3.273	3.056	2.934	2.854	0	0	0	0	0	0	0	0	0	0	
	7	3.134	2.918	2.796	2.715	2.658	0	0	0	0	0	0	0	0	0	
	8	3.035	2.820	2.697	2.616	2.558	2.515	0	0	0	0	0	0	0	0	
	9	2.960	2.746	2.623	2.541	2.483	2.440	2.405	0	0	0	0	0	0	0	
	10	2.903	2.688	2.565	2.484	2.425	2.381	2.346	2.318	0	0	0	0	0	0	
	11	2.856	2.643	2.519	2.437	2.378	2.334	2.299	2.270	2.247	0	0	0	0	0	
	12	2.819	2.605	2.481	2.399	2.340	2.295	2.260	2.231	2.207	2.187	0	0	0	0	
	13	2.787	2.574	2.450	2.367	2.308	2.263	2.227	2.198	2.174	2.154	2.137	0	0	0	
	14	2.761	2.547	2.423	2.341	2.281	2.236	2.200	2.171	2.147	2.126	2.109	2.093	0	0	
	15	2.738	2.525	2.401	2.318	2.258	2.212	2.176	2.147	2.123	2.102	2.084	2.069	2.056	0	
	16	2.719	2.505	2.381	2.298	2.238	2.192	2.156	2.126	2.102	2.081	2.063	2.048	2.034	2.022	
	17	2.701	2.488	2.364	2.280	2.220	2.174	2.138	2.108	2.083	2.062	2.045	2.029	2.015	2.003	
	18	2.686	2.473	2.348	2.265	2.204	2.158	2.122	2.092	2.067	2.046	2.028	2.012	1.999	1.986	
	19	2.673	2.459	2.335	2.251	2.191	2.144	2.108	2.078	2.053	2.032	2.013	1.998	1.984	1.971	
	20	2.661	2.447	2.323	2.239	2.178	2.132	2.095	2.065	2.040	2.019	2.000	1.984	1.970	1.958	
	21	2.650	2.437	2.312	2.228	2.167	2.121	2.084	2.053	2.028	2.007	1.988	1.972	1.958	1.946	
	22	2.640	2.427	2.302	2.218	2.157	2.110	2.073	2.043	2.018	1.996	1.978	1.962	1.947	1.935	
	23	2.631	2.418	2.293	2.209	2.148	2.101	2.064	2.033	2.008	1.987	1.968	1.952	1.938	1.925	
	24	2.623	2.410	2.285	2.201	2.139	2.092	2.055	2.025	1.999	1.978	1.959	1.943	1.928	1.916	
	25	2.616	2.402	2.277	2.193	2.132	2.085	2.047	2.017	1.991	1.969	1.951	1.934	1.920	1.907	
	26	2.609	2.396	2.270	2.186	2.125	2.078	2.040	2.009	1.984	1.962	1.943	1.927	1.912	1.900	
	27	2.602	2.389	2.264	2.180	2.118	2.071	2.033	2.003	1.977	1.955	1.936	1.920	1.905	1.892	
	28	2.597	2.383	2.258	2.174	2.112	2.065	2.027	1.996	1.971	1.949	1.930	1.913	1.899	1.886	
	29	2.591	2.378	2.252	2.168	2.106	2.059	2.021	1.990	1.965	1.943	1.924	1.907	1.893	1.880	
	30	2.586	2.373	2.247	2.163	2.101	2.054	2.016	1.985	1.959	1.937	1.918	1.901	1.887	1.874	
40	2.550	2.337	2.211	2.126	2.063	2.015	1.977	1.946	1.919	1.897	1.877	1.860	1.845	1.832		
50	2.528	2.315	2.189	2.104	2.041	1.993	1.954	1.922	1.896	1.873	1.853	1.836	1.820	1.807		
60	2.514	2.301	2.175	2.089	2.026	1.978	1.939	1.907	1.880	1.857	1.837	1.819	1.804	1.790		
70	2.504	2.291	2.164	2.079	2.016	1.967	1.928	1.896	1.869	1.846	1.825	1.808	1.792	1.778		
80	2.496	2.283	2.157	2.071	2.008	1.959	1.920	1.887	1.860	1.837	1.817	1.799	1.783	1.769		
90	2.491	2.277	2.151	2.065	2.002	1.953	1.913	1.881	1.854	1.830	1.810	1.792	1.776	1.762		
100	2.486	2.273	2.146	2.060	1.997	1.948	1.908	1.876	1.849	1.825	1.805	1.787	1.771	1.757		
125	2.478	2.264	2.138	2.051	1.988	1.939	1.899	1.867	1.839	1.816	1.795	1.777	1.761	1.747		
150	2.472	2.259	2.132	2.046	1.982	1.933	1.893	1.861	1.833	1.809	1.789	1.770	1.754	1.740		
175	2.468	2.255	2.128	2.042	1.978	1.929	1.889	1.856	1.828	1.805	1.784	1.766	1.750	1.735		
200	2.465	2.252	2.125	2.039	1.975	1.925	1.886	1.853	1.825	1.801	1.781	1.762	1.746	1.732		

Table 2-5: B-Basis factors for small datasets using variability of corresponding large dataset

### 3. Summary of Results

The basis values for all tests are summarized in the following tables. The NCAMP recommended B-basis values meet all requirements of CMH-17-1G. However, not all test data meets those requirements. The summary tables provide a complete listing of all computed basis values and estimates of basis values. Data that does not meet the requirements of CMH-17-1G are shown in shaded boxes and labeled as estimates. Basis values computed with the modified coefficient of variation (CV) are presented whenever possible. Basis values and estimates computed without that modification are presented for all tests.

#### 3.1 NCAMP Recommended B-basis Values

The following rules are used in determining what B-basis value, if any, is included in Table 3-1 of recommended values.

1. Recommended values are NEVER estimates. Only B-basis values that meet all requirements of CMH-17-1G are recommended.
2. Modified CV basis values are preferred. Recommended values will be the modified CV basis value when available. The CV provided with the recommended basis value will be the one used in the computation of the basis value.
3. Only normalized basis values are given for properties that are normalized.
4. ANOVA B-basis values are not recommended since only three batches of material are available and CMH-17-1G recommends that no less than five batches be used when computing basis values with the ANOVA method.
5. Basis values of 90% or more of the mean value imply that the CV is unusually low and may not be conservative. Caution is recommended with B-Basis values calculated using the single point method when the B-basis value is 90% or more of the average value. Such values will be indicated.
6. If the data appear questionable (e.g. when the CTD-RTD-ETW trend of the basis values are not consistent with the CTD-RTD-ETW trend of the average values), then the B-basis values will not be recommended.

**NCAMP Recommended B-basis Values for  
TenCate Advance Composites AS4C 3KPW with BT250E-6 Resin Material**  
All B-basis values in this table meet the standards for publication in CMH-17G Handbook  
Values are for normalized data unless otherwise noted

**Lamina Strength Tests**

Environment	Statistic	WT	FT	WC	FC	SBS*	IPS*	
							0.2% Offset	5% Strain
CTD (-65° F)	B-basis	108.980	107.321	88.943	82.084	7.508	7.436	11.731
	Mean	125.113	120.271	102.760	93.260	8.466	8.396	13.246
	CV	6.694	6.244	7.127	7.414	6.000	6.000	6.000
RTD (70° F)	B-basis	NA:A	111.800	81.362	73.946	7.242	5.671	9.154
	Mean	132.294	124.751	93.092	85.122	8.157	6.403	10.335
	CV	4.792	6.000	6.612	6.604	6.000	6.000	6.000
ETD (180° F)	B-basis				NA:I	6.254		
	Mean				73.543	7.109		
	CV				9.239	6.308		
ETW (180° F)	B-basis	110.945	97.437	48.906	NA:A	4.452	3.267	5.071
	Mean	124.698	110.388	56.398	50.327	5.044	3.688	5.733
	CV	6.000	6.767	6.971	7.059	6.222	6.000	6.000

Notes: The modified CV B-basis value is recommended when available.  
 The CV provided corresponds with the B-basis value. If no B-basis, then actual CV is shown.  
 NA implies that tests were run but data did not meet NCAMP recommended requirements.  
 "NA: A" indicates ANOVA with 3 batches, "NA: I" indicates insufficient data,  
 Shaded empty boxes indicate that no test data is available for that property and condition.  
 \* Data is as-measured rather than normalized  
 \*\* Derived from cross-ply using back-out factor  
 \*\*\* indicates the Stat17 B-basis value is greater than 90% of the mean value.

**Table 3-1: NCAMP recommended B-basis values for lamina test data**

### 3.2 Lamina Summary Tables

<b>Prepreg Material:</b>	TenCate Advance Composites AS4C 3k-PW Fabric with BT250E-6 Resin	<b>Lamina Properties Summary TenCate Advance Composites AS4C 3k-PW Fabric with BT250E-6 Resin</b>
<b>Material Specification:</b>	NMS 250/2	
<b>Process Specification:</b>	NPS 81250	
<b>Fiber:</b>	AS4C 3k-PW <b>Resin:</b> TenCate BT250E-6	
<b>Tg(dry):</b> 275.46°F <b>Tg(wet):</b> 249.55°F <b>Tg METHOD:</b> ASTM D7028		

<b>Date of fiber manufacture</b>	Batch 1	Batch 2	Batch 3	<b>Date of testing</b>	Nov 2011 - Apr 2012
<b>Date of resin manufacture *</b>	5/5/11	7/24/09	4/5/11	<b>Date of data submittal</b>	May-12
<b>Date of prepreg manufacture</b>	3/8/11	5/10/11	5/10/11	<b>Date of analysis</b>	Nov-15
<b>Date of composite manufacture</b>	6/13/11	5/31/11	5/31/11		
	Sept 2011 - Oct 2011				

<b>LAMINA MECHANICAL PROPERTY B-BASIS SUMMARY</b>												
Data reported: As-measured followed by normalized values in parentheses, normalizing tply: 0.0085 in												
Values shown in shaded boxes do not meet CMH17 Rev G requirements and are estimates only												
These values may not be used for certification unless specifically allowed by the certifying agency												
	CTD			RTD			ETD			ETW		
	B-Basis	Modified CV B-basis	Mean	B-Basis	Modified CV B-basis	Mean	B-Basis	Modified CV B-basis	Mean	B-Basis	Modified CV B-basis	Mean
<b>F<sub>1</sub><sup>tu</sup> (ksi)</b>	84.777 (89.686)	108.137 (108.980)	124.754 (125.113)	95.054 (96.323)	115.788 (116.331)	131.913 (132.294)				93.993 (101.390)	109.557 (110.945)	123.157 (124.698)
<b>E<sub>1</sub><sup>t</sup> (Msi)</b>			8.592 (8.631)			8.536 (8.560)						8.452 (8.558)
<b>v<sub>12</sub><sup>t</sup></b>			0.058			0.055						0.051
<b>F<sub>2</sub><sup>tu</sup> (ksi)</b>	91.105 (95.635)	106.806 (107.321)	120.324 (120.271)	114.884 (115.323)	111.805 (111.800)	125.323 (124.751)				96.457 (99.319)	96.646 (97.437)	110.164 (110.388)
<b>E<sub>2</sub><sup>t</sup> (Msi)</b>			8.615 (8.641)			8.569 (8.530)						8.585 (8.604)
<b>F<sub>1</sub><sup>cu</sup> (ksi)</b>	71.527 (76.273)	87.144 (88.943)	101.147 (102.760)	83.326 (83.827)	80.988 (81.362)	92.800 (93.092)				48.855 (50.014)	47.921 (48.906)	55.442 (56.398)
<b>E<sub>1</sub><sup>c</sup> (Msi)</b>			7.937 (8.060)			7.890 (7.905)						7.973 (7.987)
<b>F<sub>2</sub><sup>cu</sup> (ksi)</b>	83.287 (83.529)	83.609 (82.084)	93.152 (93.260)	75.375 (75.391)	75.697 (73.946)	85.240 (85.122)	55.764 (54.676)	61.132 NA	72.056 (73.543)	35.281 (30.243)	40.363 (43.178)	49.864 (50.327)
<b>E<sub>2</sub><sup>c</sup> (Msi)</b>			8.109 (8.111)			7.882 (7.862)			7.724 (7.896)			7.886 (7.906)
<b>F<sub>12</sub><sup>s5%</sup> (ksi)</b>	12.551	11.731	13.246	9.943	9.154	10.335				5.367	5.071	5.733
<b>F<sub>12</sub><sup>s0.2%</sup> (ksi)</b>	7.567	7.436	8.396	6.112	5.671	6.403				3.477	3.267	3.688
<b>G<sub>12</sub><sup>s</sup> (Msi)</b>			0.662			0.578						0.379
<b>SBS (ksi)</b>	7.990	7.708	8.466	7.682	7.402	8.157	6.630	6.347	7.109	4.568	4.286	5.044

\* Three unique resin lots were used, two of the resin lots were produced on the same day.

**Table 3-2: Summary of Test Results for Lamina Data**

#### 4. Test Results, Statistics, Basis Values, and Graphs

Test data for fiber dominated properties was normalized according to nominal cured ply thickness. Both normalized and as-measured statistics were included in the tables, but only the normalized data values were graphed. Test failures, outliers and explanations regarding computational choices were noted in the accompanying text for each test.

All individual specimen results are graphed for each test by batch and environmental condition with a line indicating the recommended basis values for each environmental condition. The data is jittered (moved slightly to the left or right) in order for all specimen values to be clearly visible. The strength values are always graphed on the vertical axis with the scale adjusted to include all data values and their corresponding basis values. The vertical axis may not include zero. The horizontal axis values will vary depending on the data and how much overlapping there was of the data within and between batches. When there was little variation, the batches were graphed from left to right. The environmental conditions were identified by the shape and color of the symbol used to plot the data. Otherwise, the environmental conditions were graphed from left to right and the batches were identified by the shape and color of the symbol.

When a dataset fails the Anderson-Darling k-sample (ADK) test for batch-to-batch variation, an ANOVA analysis is required. In order for B-basis values to be computed using the ANOVA method, data from five batches are required. Since this qualification dataset has only three batches, the basis values computed using ANOVA are considered estimates only. However, the basis values resulting from the ANOVA method using only three batches may be overly conservative. The ADK test is performed again after a transformation of the data according to the assumptions of the modified CV method (see section 2.1.4 for details). If the dataset still passes the ADK test at this point, modified CV basis values are provided. If the dataset does not pass the ADK test after the transformation, estimates may be computed using the modified CV method per the guidelines in CMH-17 Vol 1 Chapter 8 section 8.3.10.

### 4.1 Warp Tension (WT)

The Warp tension strengths are normalized. All WT datasets failed the Anderson Darling k-sample test (ADK test) for batch to batch variability, which meant pooling across environments was not acceptable. CMH-17-1G guidelines required using the ANOVA analysis. With fewer than 5 batches, this is considered an estimate. When the ETW as-measured datasets and the CTD and ETW normalized dataset were transformed according to the assumptions of the modified CV method, they passed the ADK test, so the modified CV basis values are provided. Modified CV estimates are provided for the as-measured CTD dataset and the RTD datasets, both as-measured and normalized.

There was one outlier. The lowest value in batch two of the RTD condition was an outlier for batch two as well as for the RTD condition. It was an outlier for both the as-measured and the normalized RTD datasets. It was retained for this analysis.

Statistics and basis values are given for strength data in Table 4-1 and for the modulus data in Table 4-2. The data and the B-basis values are shown graphically in Figure 4-1.

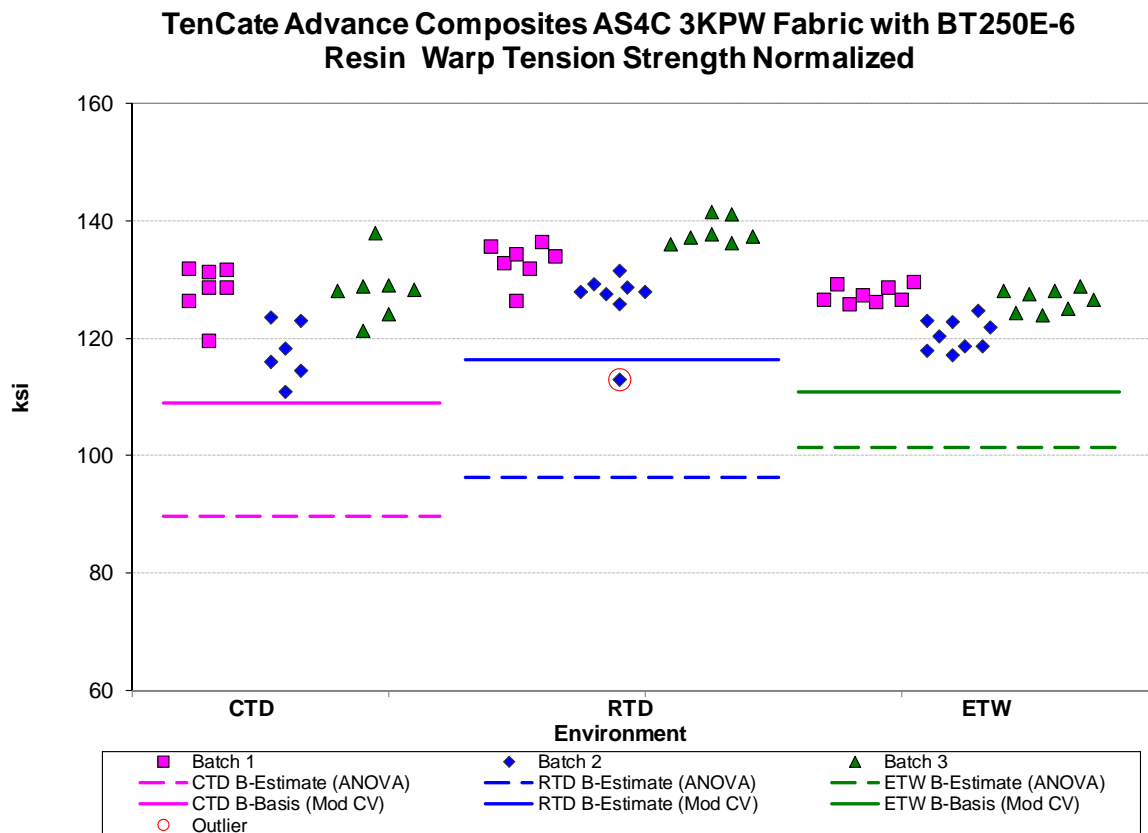


Figure 4-1: Batch plot for WT strength normalized



<b>Warp Tension Strength Basis Values and Statistics</b>						
	<b>Normalized</b>			<b>As-measured</b>		
<b>Env</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>
Mean	125.113	132.294	124.698	124.754	131.913	123.157
Stdev	6.741	6.340	3.762	7.272	6.541	4.944
CV	5.388	4.792	3.017	5.829	4.959	4.015
Mod CV	6.694	6.396	6.000	6.915	6.479	6.007
Min	110.920	112.881	117.096	110.386	112.748	113.780
Max	137.895	141.459	129.636	135.538	141.422	131.283
No. Batches	3	3	3	3	3	3
No. Spec.	20	22	25	20	22	25
B-estimate	89.686	96.323	101.390	84.777	95.054	93.993
A-estimate	64.402	70.640	84.744	56.245	68.736	73.161
Method	ANOVA	ANOVA	ANOVA	ANOVA	ANOVA	ANOVA
<b>Modified CV Basis Values and Estimates</b>						
B-basis Value	108.980		110.945			109.557
B-estimate		116.331		108.137	115.788	
A-estimate	97.514	104.935	101.070	96.327	104.277	99.793
Method	Normal	Normal	Normal	Normal	Normal	Normal

Table 4-1: Statistics and Basis values for WT strength

<b>Warp Tension Modulus Statistics</b>						
	<b>Normalized</b>			<b>As-measured</b>		
<b>Env</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>
Mean	8.631	8.560	8.558	8.592	8.536	8.452
Stdev	0.085	0.057	0.085	0.215	0.188	0.179
CV	0.981	0.663	0.994	2.504	2.205	2.118
Mod CV	6.000	6.000	6.000	6.000	6.000	6.000
Min	8.485	8.468	8.411	8.257	8.212	8.057
Max	8.829	8.698	8.747	9.004	8.911	8.715
No. Batches	3	3	3	3	3	3
No. Spec.	18	22	28	18	22	28

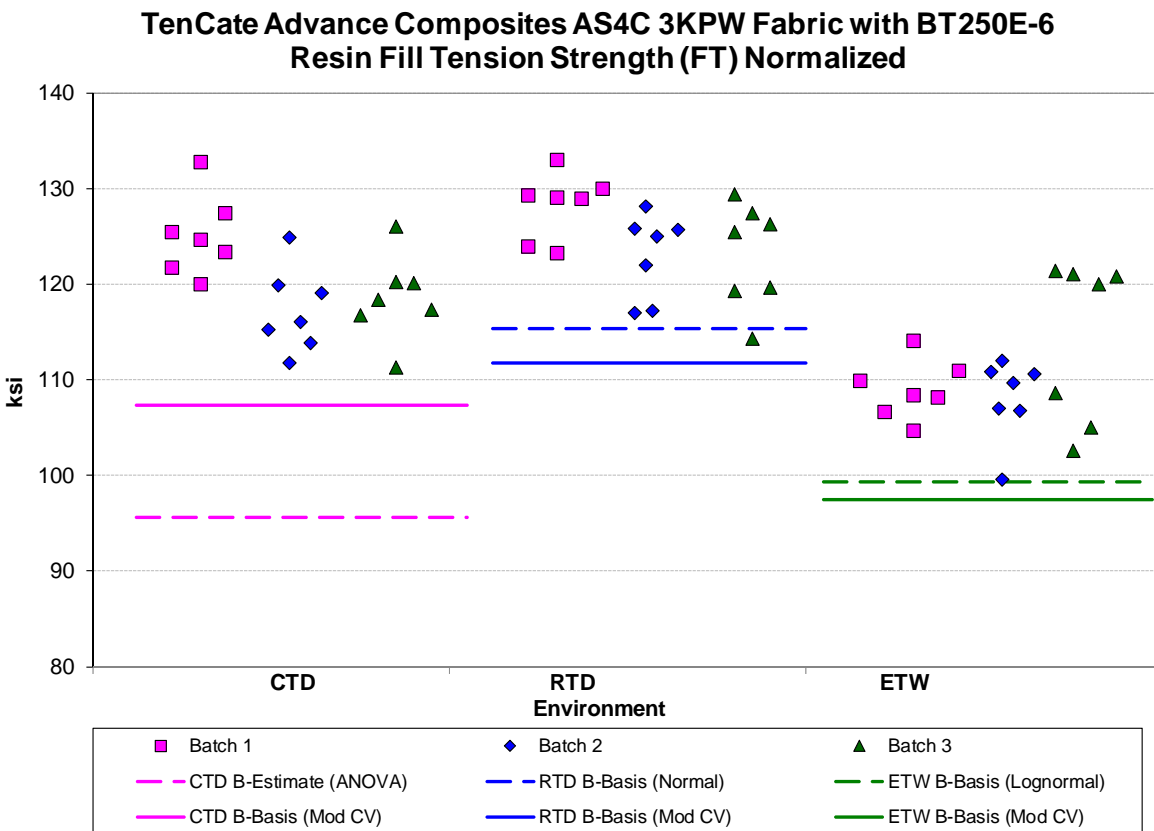
Table 4-2: Statistics from WT modulus

### 4.2 Fill Tension (FT)

Fill Tension data was normalized. The CTD dataset, both normalized and as-measured, failed the Anderson Darling k-sample test (ADK test) for batch to batch variability, which meant the CTD condition required using the ANOVA analysis according to CMH-17-1G guidelines. With fewer than 5 batches, this is considered an estimate. The CTD dataset passed the ADK test after applying the modified CV transformation to the data, thus modified CV results are available. Pooling the three environments was acceptable for the modified CV basis value computations.

There were two outliers, the largest value in batch one of the CTD condition and the lowest value in batch two of the ETW condition. Both outliers were outliers only for the as-measured data, not for the normalized data, and both were outliers only for their respective batches, not for their respective conditions. Both outliers were retained for this analysis.

Statistics, basis values and estimates are given for strength data as-measured in Table 4-3 and for the modulus data as-measured in Table 4-4. The data and the B-basis values and B-estimates are shown graphically in Figure 4-2.



**Figure 4-2: Batch Plot for FT strength Normalized**

<b>Fill Tension Strength Basis Values and Statistics</b>						
Normalized				As-measured		
Env	CTD	RTD	ETW	CTD	RTD	ETW
Mean	120.271	124.751	110.388	120.324	125.323	110.164
Stdev	5.398	4.949	6.108	6.250	5.480	6.699
CV	4.488	3.967	5.533	5.194	4.373	6.081
Mod CV	6.244	6.000	6.767	6.597	6.186	7.040
Min	111.238	114.261	99.509	109.120	114.351	98.594
Max	132.730	133.012	121.328	133.218	132.998	122.561
No. Batches	3	3	3	3	3	3
No. Spec.	21	21	21	21	21	21
<b>Basis Values and Estimates</b>						
B-basis Value		115.323	99.319		114.884	96.457
B-estimate	95.635			91.105		
A-estimate	78.050	108.602	92.206	70.248	107.441	74.123
Method	ANOVA	Normal	Lognormal	ANOVA	Normal	Non-Parametric
<b>Modified CV Basis Values and Estimates</b>						
B-basis Value	107.321	111.800	97.437	106.806	111.805	96.646
A-estimate	98.578	103.058	88.695	97.681	102.680	87.521
Method	pooled	pooled	pooled	pooled	pooled	pooled

Table 4-3: Statistics and Basis Values for FT Strength data

<b>Fill Tension Modulus Statistics</b>						
Normalized				As-measured		
Env	CTD	RTD	ETW	CTD	RTD	ETW
Mean	8.641	8.530	8.604	8.615	8.569	8.585
Stdev	0.088	0.118	0.111	0.084	0.170	0.139
CV	1.021	1.385	1.286	0.978	1.983	1.618
Mod CV	6.000	6.000	6.000	6.000	6.000	6.000
Min	8.482	8.215	8.421	8.404	8.195	8.363
Max	8.783	8.737	8.790	8.746	9.003	8.871
No. Batches	3	3	3	3	3	3
No. Spec.	18	21	21	18	21	21

Table 4-4: Statistics from FT Modulus data

### 4.3 Warp Compression (WC)

The warp compression data was normalized. The CTD dataset, both normalized and as-measured, failed the Anderson Darling k-sample test (ADK test) for batch to batch variability, which meant the CTD condition required using the ANOVA analysis according to CMH-17-1G guidelines. With fewer than 5 batches, this is considered an estimate. The CTD dataset passed the ADK test after applying the modified CV transformation to the data, thus modified CV results are available. Pooling the three environments was not acceptable for the modified CV basis value computations due to the pooled dataset failing the normality test.

There was one outlier, the lowest value in batch three of the RTD condition. It was an outlier only for the as-measured data, not for the normalized data. It was an outlier only for batch three, not for the RTD condition. It was retained for this analysis.

Statistics and B-estimates are given for strength data in Table 4-5 and for the modulus data in Table 4-6. The data and the B-estimates are shown graphically in Figure 4-3.

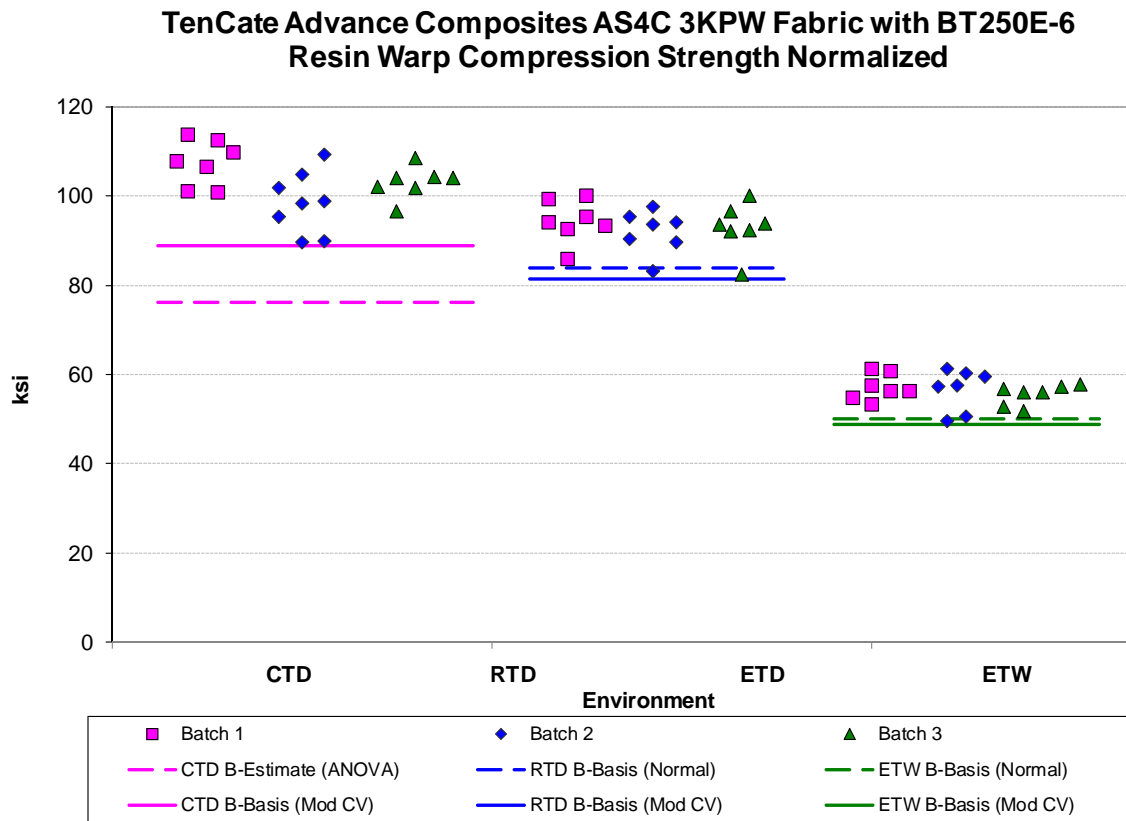


Figure 4-3: Batch plot for WC strength normalized

<b>Warp Compression Strength Basis Values and Statistics</b>						
	<b>Normalized</b>			<b>As-measured</b>		
<b>Env</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>
Mean	102.760	93.092	56.398	101.147	92.800	55.442
Stdev	6.426	4.863	3.351	6.753	4.974	3.457
CV	6.254	5.224	5.942	6.677	5.359	6.236
Mod CV	7.127	6.612	6.971	7.338	6.680	7.118
Min	89.515	82.364	49.677	87.625	81.462	47.952
Max	113.672	100.158	61.373	112.219	101.045	60.572
No. Batches	3	3	3	3	3	3
No. Spec.	22	21	21	22	21	21
<b>Basis Value Estimates</b>						
B-basis Value		83.827	50.014		83.326	48.855
B-estimate	76.273			71.527		
A-Estimate	57.361	77.222	45.462	50.376	76.571	44.160
Method	ANOVA	Normal	Normal	ANOVA	Normal	Normal
<b>Modified CV Basis Value Estimates</b>						
B-basis Value	88.943	81.362	48.906	87.144	80.988	47.921
A-Estimate	79.080	73.007	43.569	77.147	72.574	42.565
Method	Normal	Normal	Normal	Normal	Normal	Normal

Table 4-5: Statistics and Basis Values for WC strength

<b>Warp Compression Modulus Statistics</b>						
	<b>Normalized</b>			<b>As-measured</b>		
<b>Env</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>	<b>CTD</b>	<b>RTD</b>	<b>ETW</b>
Mean	8.060	7.905	7.987	7.937	7.890	7.973
Stdev	0.127	0.081	0.141	0.121	0.126	0.183
CV	1.572	1.019	1.760	1.519	1.593	2.294
Mod CV	6.000	6.000	6.000	6.000	6.000	6.000
Min	7.819	7.783	7.622	7.699	7.690	7.550
Max	8.391	8.067	8.172	8.214	8.106	8.282
No. Batches	3	3	3	3	3	3
No. Spec.	18	18	21	18	18	21

Table 4-6: Statistics from WC modulus

#### 4.4 Fill Compression (FC)

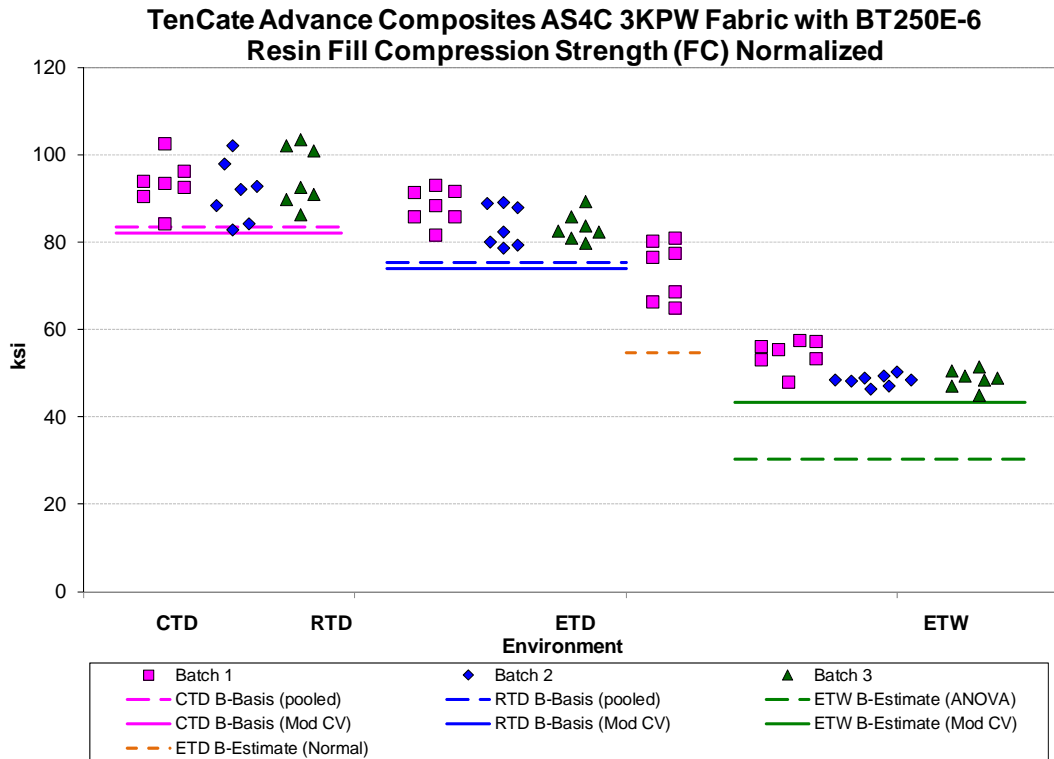
Fill Compression data is normalized. The ETD dataset has results from only seven specimens, insufficient for CMH17 requirements, so only estimates are provided for that condition. The CTD and RTD conditions met all requirements for pooling, so that method was used to compute basis values.

The ETW datasets, both normalized and as measured, failed the Anderson Darling k-sample test (ADK test) for batch to batch variability, which means that pooling across environments was not acceptable and CMH-17-1G guidelines required using the ANOVA analysis. With fewer than 5 batches, this is considered an estimate. When the ETW datasets were transformed according to the assumptions of the modified CV method, only the as-measured ETW dataset passed the ADK test, so the modified CV basis values are provided for the as-measured ETW dataset while only estimates are available for the normalized ETW dataset. The as-measured pooled dataset passed all diagnostic tests, so pooling across all four environmental conditions was acceptable for the modified CV basis values computations.

No modified CV basis values are provided for the normalized ETD condition due to the CV being above 8% and pooling across the environments was unacceptable due to the failure of the normalized ETW dataset to pass the ADK test even after the modified CV transformation.

There were no outliers.

Statistics, basis values and estimates are given for strength data in Table 4-7 and for the modulus data in Table 4-8. The data, B-estimates, and B-basis values are shown graphically in Figure 4-4.



**Figure 4-4: Batch Plot for FC strength Normalized**

Fill Compression Strength Basis Values and Statistics									
Env	Normalized				As-measured				
	CTD	RTD	ETD	ETW	CTD	RTD	ETD	ETW	
Mean	93.260	85.122	73.543	50.327	93.152	85.240	72.056	49.864	
Stdev	6.368	4.433	6.795	3.553	6.701	4.118	5.868	3.007	
CV	6.828	5.207	9.239	7.059	7.194	4.832	8.143	6.029	
Mod CV	7.414	6.604	9.239	7.530	7.597	6.416	8.143	7.015	
Min	82.771	78.582	64.866	44.995	82.690	79.502	64.293	45.571	
Max	103.399	93.019	81.002	57.347	104.268	92.628	78.115	55.730	
No. Batches	3	3	1	3	3	3	1	3	
No. Spec.	21	21	7	22	21	21	7	22	
Basis Values and Estimates									
B-basis Value	83.529	75.391			83.287	75.375			
B-estimate			54.676	30.243			55.764	35.281	
A-estimate	76.841	68.703	41.404	15.902	76.507	68.595	44.303	24.868	
Method	pooled	pooled	Normal	ANOVA	pooled	pooled	Normal	ANOVA	
Modified CV Basis Values and Estimates									
B-basis Value	82.084	73.946	NA		83.609	75.697		40.363	
B-estimate					43.178			61.132	
A-estimate	74.403	66.264			38.075	77.190	69.279	54.875	33.937
Method	pooled	pooled			Normal	pooled	pooled	pooled	pooled

**Table 4-7: Statistics and Basis Values for FC Strength data**

Fill Compression Modulus Statistics								
	Normalized				As-measured			
Env	CTD	RTD	ETD	ETW	CTD	RTD	ETD	ETW
Mean	8.111	7.862	7.896	7.906	8.109	7.882	7.724	7.886
Stdev	0.177	0.178	0.263	0.097	0.233	0.211	0.135	0.120
CV	2.181	2.269	3.329	1.222	2.873	2.676	1.751	1.521
Mod CV	6.000	6.000	6.000	6.000	6.000	6.000	6.000	6.000
Min	7.688	7.576	7.629	7.737	7.691	7.661	7.555	7.682
Max	8.439	8.450	8.157	8.101	8.526	8.596	7.868	8.065
No. Batches	3	3	1	3	3	3	1	3
No. Spec.	18	18	6	21	18	18	6	21

Table 4-8: Statistics from FC Modulus data



### 4.5 In-Plane Shear (IPS)

In Plane Shear data is not normalized. The 0.2% offset strength dataset for the CTD condition failed the Anderson Darling k-sample test (ADK test) for batch to batch variability, which means that pooling across environments was not acceptable and CMH-17-1G guidelines required using the ANOVA analysis. With fewer than 5 batches, this is considered an estimate. When the dataset was transformed according to the assumptions of the modified CV method, both passed the ADK test, so the modified CV basis values are provided. Pooling was not acceptable for the combined datasets due to the pooled dataset failing Levene’s test for equality of variance.

There were no outliers.

Statistics, estimates and basis values are given for the strength data in Table 4-9 and modulus data in Table 4-10. The data, B-estimates and B-basis values are shown graphically in Figure 4-5.

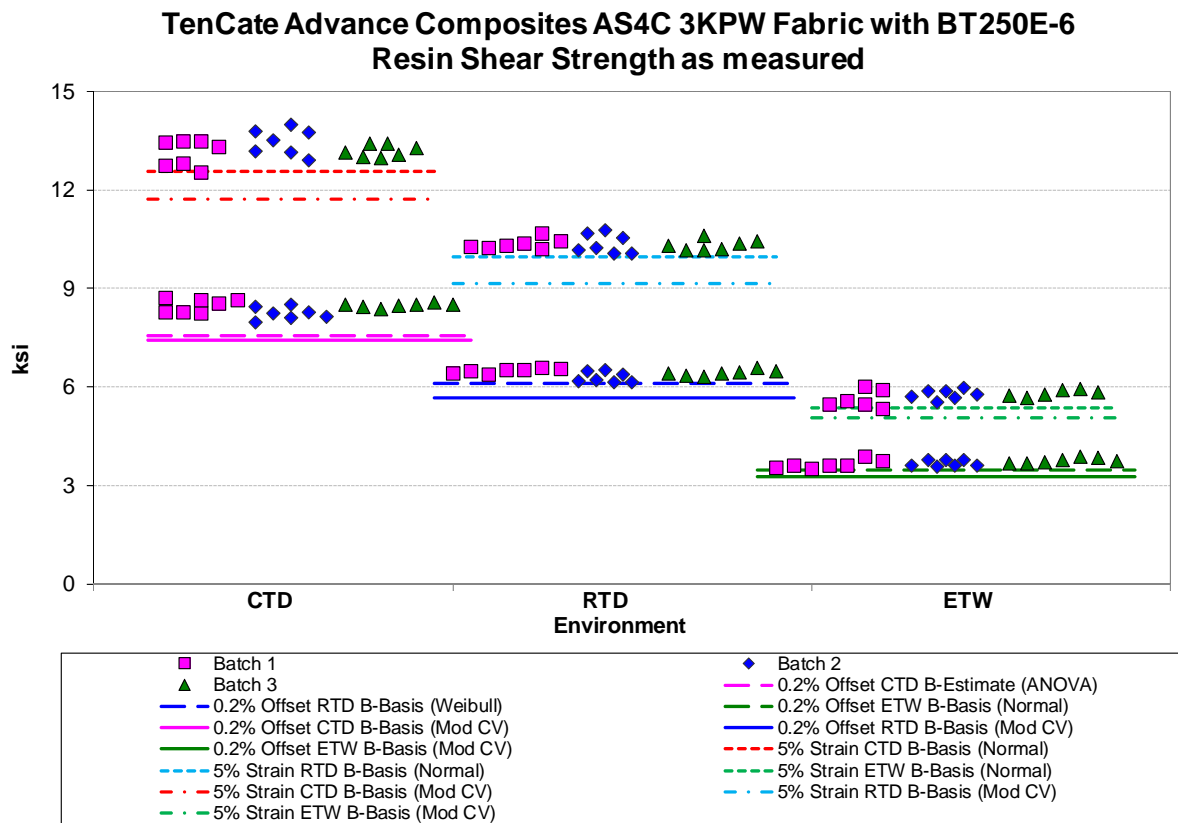


Figure 4-5: Batch plot for IPS for 0.2% offset strength and strength at 5% strain as-measured

<b>In-Plane Shear Strength Basis Values and Statistics</b>						
	<b>Strength at 5% Strain</b>			<b>0.2% Offset Strength</b>		
Env	CTD	RTD	ETW	CTD	RTD	ETW
Mean	13.246	10.335	5.733	8.396	6.403	3.688
Stdev	0.365	0.206	0.190	0.197	0.141	0.111
CV	2.753	1.991	3.319	2.348	2.207	3.004
Mod CV	6.000	6.000	6.000	6.000	6.000	6.000
Min	12.517	10.050	5.337	7.957	6.134	3.497
Max	13.973	10.779	6.010	8.724	6.583	3.873
No. Batches	3	3	3	3	3	3
No. Spec.	21	21	20	21	21	21
<b>Basis Values and Estimates</b>						
B-basis Value	12.551	9.943	5.367		6.112	3.477
B-estimate				7.567		
A-estimate	12.056	9.664	5.106	6.975	5.789	3.327
Method	Normal	Normal	Normal	ANOVA	Weibull	Normal
<b>ModifiedBasis Values and Estimates</b>						
B-basis Value	11.731	9.154	5.071	7.436	5.671	3.267
A-estimate	10.652	8.312	4.600	6.752	5.149	2.966
Method	Normal	Normal	Normal	Normal	Normal	Normal

Table 4-9: Statistics and Basis Values for IPS Strength data

<b>In Plane Shear Modulus Statistics</b>			
Env	CTD	RTD	ETW
Mean	0.662	0.578	0.379
Stdev	0.016	0.014	0.010
CV	2.460	2.387	2.760
Mod CV	6.000	6.000	6.000
Min	0.629	0.550	0.364
Max	0.692	0.597	0.398
No. Batches	3	3	3
No. Spec.	21	21	21

Table 4-10: Statistics from IPS Modulus data

### 4.6 Lamina Short-Beam Strength (SBS)

The Short Beam Strength data is not normalized. There were no diagnostic test failures, so pooling the four environmental conditions was acceptable. However, after transforming the data to meet the requirements of the modified CV method, pooling across environments was not acceptable for the combined datasets due to failing Levene’s test for equality of variance. There were no outliers.

Statistics and basis values are given for SBS data in Table 4-11. The data and the B-basis values are shown graphically in Figure 4-6.

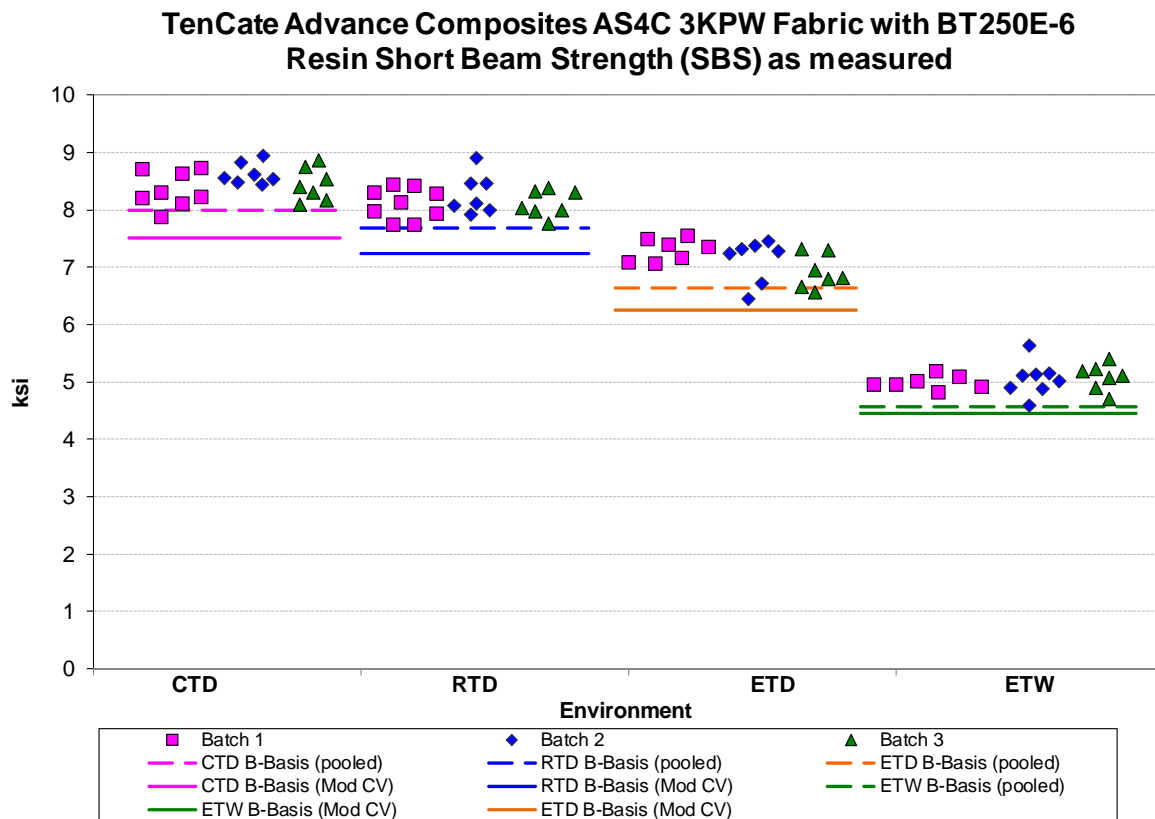


Figure 4-6: Batch plot for SBS as-measured

<b>Short Beam Strength Basis Values and Statistics</b>				
<b>As-measured</b>				
<b>Env</b>	<b>CTD</b>	<b>RTD</b>	<b>ETD</b>	<b>ETW</b>
<b>Mean</b>	8.466	8.157	7.109	5.044
<b>Stdev</b>	0.281	0.282	0.328	0.224
<b>CV</b>	3.323	3.458	4.617	4.444
<b>Mod CV</b>	6.000	6.000	6.308	6.222
<b>Min</b>	7.885	7.745	6.443	4.597
<b>Max</b>	8.948	8.896	7.554	5.630
<b>No. Batches</b>	3	3	3	3
<b>No. Spec.</b>	22	23	21	22
<b>Basis Values and Estimates</b>				
<b>B-estimate</b>	7.990	7.682	6.630	4.568
<b>A-estimate</b>	7.671	7.363	6.311	4.249
<b>Method</b>	pooled	pooled	pooled	pooled
<b>Modified CV Basis Values and Estimates</b>				
<b>B-estimate</b>	7.508	7.242	6.254	4.452
<b>A-estimate</b>	6.824	6.588	5.645	4.030
<b>Method</b>	Normal	Normal	Normal	Normal

**Table 4-11: Statistics and Basis Values for SBS data**

## 5. Outliers

Outliers were identified according to the standards documented in section 2.1.5, which are in accordance with the guidelines developed in section 8.3.3 of CMH-17-1G. An outlier may be an outlier in the normalized data, the as-measured data, or both. A specimen may be an outlier for the batch only (before pooling the three batches within a condition together) or for the condition (after pooling the three batches within a condition together) or both.

Approximately 5 out of 100 specimens will be identified as outliers due to the expected random variation of the data. This test is used only to identify specimens to be investigated for a cause of the extreme observation. Outliers that have an identifiable cause are removed from the dataset as they inject bias into the computation of statistics and basis values. Specimens that are outliers for the condition and in both the normalized and as-measured data are typically more extreme and more likely to have a specific cause and be removed from the dataset than other outliers. Specimens that are outliers only for the batch, but not the condition and specimens that are identified as outliers only for the normalized data or the as-measured data but not both, are typical of normal random variation.

All outliers identified were investigated to determine if a cause could be found. Outliers with causes were removed from the dataset and the remaining specimens were analyzed for this report. Information about specimens that were removed from the dataset along with the cause for removal is documented in the material property data report, NCAMP Test Report CAM-RP-2015-039 Rev N/C.

Outliers for which no causes could be identified are listed in Table 5-1. These outliers were included in the analysis for their respective test properties.

Test	Condition	Batch	Specimen Number	Normalized Strength	Strength As-measured	High/Low	Batch Outlier	Condition Outlier
WC	RTD	C	EABLC113A	Not an Outlier	81.462	Low	Yes	No.
WT	RTD	B	EABJB111A	112.881	112.748	Low	Yes	Yes
FT	CTD	A	EABUA116B	Not an Outlier	133.218	High	Yes	No.
FT	ETW	B	EABUB11ED	Not an Outlier	98.594	Low	Yes	No.

**Table 5-1: List of Outliers**

## 6. References

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